King Saud University
College of Sciences
Department of Mathematics

First Mid Term Exam, S2 1445
M 380 – Stochastic Processes
Time: 90 minutes

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Answer all questions:

Q1: [2.5+2.5]

(a) For $Y \sim Poisson(\lambda)$, determine the probability that Y is even.

Hint: use the series expansion of $\cosh \lambda = \frac{1}{2}(e^\lambda + e^{-\lambda}) = 1 + \frac{\lambda^2}{2!} + \frac{\lambda^4}{4!} + \frac{\lambda^6}{6!} + \ldots$

(b) Prove that an exponential random variable X with parameter λ satisfies the memoryless property, and use that to find:

$Pr(X \leq 100 | X > 90)$ for $X \sim exp(0.04)$.

Q2: [5]

Suppose that the price X of a particular stock at closing has a log-normal distribution with mean \$25 and a variance 5. What is the probability that the price exceeds \$30?

Q3: [2.5 +2.5]

(a) A fraction $p = 0.04$ of the items coming off of a production process are defective. The output of the process is sampled, in a random manner. What is the probability that the first defective item found is the fifth item sampled?

(b) The number of accidents occurring in a factory in a week is a Poisson random variable with parameter 3. The number of individuals injured in different accidents is independently distributed, each with mean 2 and variance 5. Determine the mean and variance of the number of individuals injured in a week.

Q4: [5]

(a) A loss random variable is uniformly distributed on (0,2000).
   Find the mean and variance if a policy limit of 300 **and** a deductible of 300 is
   imposed.

(b) An insurance policy reimburses a loss up to a benefit limit of 20. The policyholder's
   loss, X, follows a distribution with density function:

$$f(x) = \begin{cases} e^{-x} & , \quad x > 1, \\ 0, & otherwise \end{cases}$$

Find the expected value of the benefit paid under the insurance policy.

Q5: [5]

Let X and Y be two random variables with joint density function

$$f_{XY}(x,y) = \begin{cases} c\, e^{-y} & , \quad 0 \le x \le y < \infty, \\ 0, & otherwise \end{cases}$$

(a) Determine the value of c
(b) Find the marginal densities of X and Y
(c) Are X and Y independent? why?
(d) Find $E(Y|X = x)$.
(e) Find $E(X|Y = y)$.

Table 5.1 Area $\Phi(x)$ Under the Standard Normal Curve to the Left of X.

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998