King Saud University
College of Sciences
Department of Mathematics

Final Exam, S2 1445 H.
Math 380 – Stochastic Processes
Time: 3 hours – **Max Marks: 40**

Answer all questions.

Q1: 8 pts [4+4]

(a) An insurance policy is written to cover a loss X, where X has a uniform distribution on [0, 2000]. At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

(b) A loss random variable has density function $f(x) = 1 - x$, for $0 \le x \le 1$. At what level should a policy limit be set so that the expected insurer payment is one-half of the overall expected loss?

Hint: recall the quadratic solution formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ for $ax^2 + bx + c = 0$.

Q2: 4 pts [2+2]

(a) A Markov chain $X_0, X_1, X_2, \ldots.$ on states 0, 1, 2 has the transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{Vmatrix} 0.5 & 0.2 & 0.3 \\ 0.5 & 0.1 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{Vmatrix} \end{array}$$

and initial distributions $p_0 = Pr\{X_0 = 0\} = 0.3$, $p_1 = Pr\{X_0 = 1\} = 0.3$, and $p_2 = Pr\{X_0 = 2\} = 0.3$. Determine $Pr\{X_0 = 1, X_1 = 0, X_2 = 2\}$.

(b) Let X_n denote the quality of the n^th item produced by a production system with $X_n = 0$ meaning "good" and $X_n = 1$ meaning "defective". Suppose that X_n evolves as a Markov chain whose transition probability matrix is:

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{Vmatrix} 0.4 & 0.6 \\ 0.7 & 0.3 \end{Vmatrix} \end{array}$$

What is the probability that the fifth item is defective given that the first item is good?

Q3: 10 pts [6+4]

(a) An airline reservation system has two computers, only one of which is in operation at any given time. A computer may break down on any given day with probability **0.3**. It takes 2 days to restore a computer to normal. There is a **duplicate** repair facility so that both computers can be repaired at the same time. Form a Markov chain by taking states as the pairs (*x*, *y*), where *x* is the number of machines in operating condition at the end of a day, and *y* is 1 if a day's labour has been expended on a machine not yet repaired, and 0 otherwise.

(i) Find the transition probability matrix.

(ii) Find the probability in the long run that neither computer is operating.

(iii) What is the availability that at least one computer is operating in the long run?

(b) Find the mean time to reach state 3 starting from state 0 for the Markov chain whose transition probability matrix is:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \left\| \begin{array}{cccc} 0.3 & 0.4 & 0.1 & 0.2 \\ 0 & 0.2 & 0.1 & 0.7 \\ 0 & 0 & 0.1 & 0.9 \\ 0 & 0 & 0 & 1 \end{array} \right\| \end{array}$$

Q4: 9 pts [5+4]

(a) Suppose that customers arrive at a facility according to a Poisson process having rate $\lambda = 3$. Let $X(t)$ be the number of customers that have arrived up to time t. Determine the following probabilities:

(i) $Pr\{X(2) = 3\}$.

(ii) $Pr\{X(2) = 3 \text{ and } X(4) = 7\}$.

(iii) $Pr\{X(4) = 7 \mid X(2) = 3\}$.

(b) A pure birth process starting from $X(0) = 0$ has birth parameters $\lambda_0 = 2$, $\lambda_1 = 4$, $\lambda_2 = 3$ and $\lambda_3 = 6$. Determine $P_n(t)$ for $n = 0, 1, 2$.

Q5: 9 pts [2+3+4]

(a) For a Brownian motion $B(t)$, evaluate $Pr\{B(8) \leq 6 \mid B(0) = 2\}$.

(b) For a Brownian motion $B(t)$ with $B(0) = 0$ and for any $s, t > 0$, show that $Cov[B(t), B(s)] = min(t, s)$.

(c) Define a martingale for a continuous time process, and show that if $B(t)$ is a Brownian motion , then the process:

$$X(t) = B^2(t) - t$$

is a martingale.

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of Pr(Z<z)							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995