Q1

ⓐ $Pr\{X_1=1\} = Pr(X_1=1 \mid X_0=0)\,Pr(X_0=0) + Pr(X_1=1 \mid X_0=1)\,Pr(X_0=1)$
$+ Pr(X_1=1 \mid X_0=2)\,Pr(X_0=2)$

$= P_{01}\,P_0 + P_{11}P_1 + P_{21}P_2$

$= 0.3(0.3) + 0.2(0.5) + 0.3(0.2) = 0.25$

ⓑ $Pr(X_0=1, X_1=1, X_2=0) = P_1\,P_{11}\,P_{10}$, $P_1 = Pr(X_0=1)$

$= (0.5)(0.2)(0.4)$

$= 0.04$

$Pr(X_1=1, X_2=1, X_3=0) = P_1\,P_{11}\,P_{10}$, $P_1 = Pr(X_1=1)$

$= (0.25)(0.2)(0.4)$

$= 0.02$

ⓒ $P_2 = Pr(X_0=2) = 1$

So, $Pr(X_0=2, X_1=0, X_2=1) = P_2\,P_{20}\,P_{01}$

$= 1(0.5)(0.3) = 0.15$

Q2

ⓐ $P_r\{X_2 = 1 \mid X_0 = 1\} = P_{11}^{(2)} = \begin{bmatrix} 0.12 & 0.88 \end{bmatrix} \begin{bmatrix} 0.01 \\ 0.88 \end{bmatrix}$

$= 0.7756 = 77.56\%$

ⓑ

$$P = \begin{array}{c|cccccc} & -2 & -1 & 0 & 1 & 2 & 3 \\ \hline -2 & 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ -1 & 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 0 & 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \\ 1 & 0.2 & 0.3 & 0.4 & 0.1 & 0 & 0 \\ 2 & 0 & 0.2 & 0.3 & 0.4 & 0.1 & 0 \\ 3 & 0 & 0 & 0.2 & 0.3 & 0.4 & 0.1 \end{array}$$

using

$$P_{ij} = P_r\{X_{n+1} = j \mid X_n = i\}$$

$$= \begin{cases} P_r(\xi_{n+1} = 3-j) & , i \leq 0 \text{ replenishment} \\ P_r(\xi_{n+1} = i-j) & , 0 < i \leq 3 \text{ no replenishment} \end{cases}$$

(3)

Q3

$$u_n = \sum_{k=0}^{\infty} P_k (u_{n-1})^k \quad , n=1,2,\ldots$$

here $k=0$ or $k=2$

so
$$u_n = P_0 (u_{n-1})^0 + P_2 (u_{n-1})^2$$
$$= \frac{1}{3} + \frac{2}{3} (u_{n-1})^2$$

$$\boxed{u_n = \frac{1+2(u_{n-1})^2}{3} \quad , n=1,2,3,4,5}$$

$n=1 \Rightarrow u_1 = \frac{1}{3} = 0.3333$

$n=2 \Rightarrow u_2 = \frac{1+2(0.3333)^2}{3} = 0.4074$

$n=3 \Rightarrow u_3 = \frac{1+2(0.4074)^2}{3} = 0.44398$

$n=4 \Rightarrow u_4 = \frac{1+2(0.44398)^2}{3} = 0.4647$

$n=5 \Rightarrow u_5 = \frac{1+2(0.4647)^2}{3} = 0.4773$

Q4 (4)

(i) $u_i = Pr\{X_T = 0 \mid X_0 = i\}$ $i = 1, 2$

$v_i = E[T \mid X_0 = i]$ $i = 1, 2$

$u_1 = P_{10} + P_{11}u_1 + P_{12}u_2$

$u_2 = P_{20} + P_{21}u_1 + P_{22}u_2$

$$\Rightarrow \left.\begin{aligned} u_1 &= 0.1 + 0.6u_1 + 0.1u_2 \\ u_2 &= 0.2 + 0.3u_1 + 0.4u_2 \end{aligned}\right\} \Rightarrow \begin{aligned} 4u_1 - u_2 &= 1 \\ 3u_1 - 6u_2 &= -2 \end{aligned}$$

$$\Rightarrow u_1 = \frac{8}{21} \ \& \ \boxed{u_2 = \frac{11}{21}} \approx 0.52$$

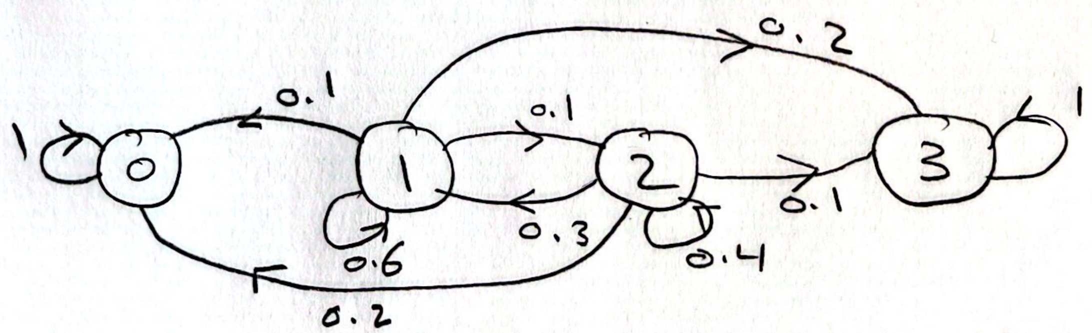
(ii) $v_1 = 1 + P_{11}v_1 + P_{12}v_2$

$v_2 = 1 + P_{21}v_1 + P_{22}v_2$

$$\Rightarrow \left.\begin{aligned} v_1 &= 1 + 0.6v_1 + 0.1v_2 \\ v_2 &= 1 + 0.3v_1 + 0.4v_2 \end{aligned}\right\} \Rightarrow \begin{aligned} 4v_1 - v_2 &= 10 \\ 3v_1 - 6v_2 &= -10 \end{aligned}$$

$$\Rightarrow v_1 = \boxed{v_2 = \frac{10}{3}} \approx 3.3$$

(iii) It is an absorbing Markov chain



Q5

$$\pi_j = \sum_{k=0}^{2} \pi_k P_{kj}$$

$j=0 \Rightarrow \pi_0 = 0.5\pi_0 + 0.5\pi_1 + 0.3\pi_2$

$\therefore 5\pi_0 - 5\pi_1 - 3\pi_2 = 0 \quad (1)$

$j=1 \Rightarrow \pi_1 = 0.2\pi_0 + 0.1\pi_1 + 0.2\pi_2$

$\therefore 2\pi_0 - 9\pi_1 + 2\pi_2 = 0 \quad (2)$

& use $\pi_0 + \pi_1 + \pi_2 = 1 \quad (3)$

Solve (1), (2) & (3) to get:

$\pi_0 = 0.4205$, $\pi_2 = 0.1818$ & $\pi_3 = 0.3977$

The long run mean cost per unit period is:

$$C = \sum_{j=0}^{2} \pi_j C_j$$

$= (0.4205)(5) + (0.1818)(8) + (0.3977)(6)$

$= \$\ 5.9431$

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4b Bonus

$$P_{GG} = P_r\{X_{n+1} = G \mid X_n = G\} = 0.4$$

$$P_{GD} = P_r\{X_{n+1} = D \mid X_n = G\} = 0.6$$

$$P_{DD} = P_r\{X_{n+1} = D \mid X_n = D\} = 0.2$$

$$P_{DG} = P_r\{X_{n+1} = G \mid X_n = D\} = 0.8$$

$$P_r\{X_1 = G, X_2 = G, X_3 = D \mid X_0 = G\}$$

$\downarrow$ Conditional property

$$= P_r\{X_3 = D \mid X_1 = G, X_2 = G, X_0 = G\} \cdot P_r\{X_1 = G, X_2 = G \mid X_0 = G\}$$

$\downarrow$ Markov property

$$= P_r\{X_3 = D \mid X_2 = G\} \cdot P_r\{X_1 = G, X_2 = G \mid X_0 = G\}$$

$\downarrow$ Similarly repeat the process

$$= P_{GD} \cdot P_r\{X_2 = G \mid X_1 = G, X_0 = G\} \cdot P_r\{X_1 = G \mid X_0 = G\}$$

$$= P_{GD} \cdot P_r\{X_2 = G \mid X_1 = G\} \cdot P_r\{X_1 = G \mid X_0 = G\}$$

$$= P_{GD} \cdot P_{GG} \cdot P_{GG}$$

$$= (0.6)(0.4)^2$$

$$= 0.096$$