

Q₁

$$P(A) = 50\%$$

$$P(D|A) = 1\%$$

(a)

$$P(B) = 35\%$$

$$P(D|B) = 2\%$$

$$P(C) = 15\%$$

$$P(D|C) = 3\%$$

$$P(D) = P(A)P(D|A) + P(B)P(D|B) + P(C)P(D|C)$$

$$= (0.50)(0.01) + (0.35)(0.02) + (0.15)(0.03)$$

$$= 0.005 + 0.007 + 0.0045$$

$$= 0.0165$$

(b)

$$P(A|D) = \frac{P(A \cap D)}{P(D)}$$

$$= \frac{0.005}{0.0165} = 0.303$$

Q₂

(a)

$$X \sim \text{Poisson}(\lambda)$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots$$

$$M_X(t) = E[e^{tx}] = \sum_{x=0}^{\infty} e^{tx} \cdot \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\begin{aligned} e^y &= \sum_{x=0}^{\infty} \frac{y^x}{x!} \quad \text{by} \\ &= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t \lambda)^x}{x!} \\ &= e^{-\lambda} e^{e^t \lambda} = e^{\lambda(e^t - 1)} \end{aligned}$$

$$M'_x(t) = \frac{d}{dt} [e^{\lambda(e^t - 1)}] = \lambda e^t e^{\lambda(e^t - 1)}$$

$$\Rightarrow E[X] = M'_x(0) = \lambda e^0 e^{\lambda(e^0 - 1)} = \lambda$$

⑥

$$\lambda = 3$$

$X = \text{no. of claims} \sim \text{Poisson}(3)$

$$P(X \geq 4) = ?$$

$$P(X \geq 4) = 1 - P(X < 4)$$

$$= 1 - P(X \leq 3)$$

$$= 1 - \{ P(X=0) + P(X=1) + P(X=2) + P(X=3) \}$$

$$\text{use } P(X=k) = \frac{e^{-\lambda} \lambda^k}{k!}, \lambda=3$$

$$= 1 - 13e^{-3}$$

Q3

$X \sim \text{log-normal dist.}, P(X > 35)$

$$E[X] = 30 \quad \text{Var}[X] = 5$$

$$P(X > 35) = 1 - P(X \leq 35)$$

$$= 1 - P(\ln X \leq \ln 35)$$

Find μ and σ :

$$E[X] = e^{\mu + \sigma^2/2} = 30 \rightarrow (1)$$

$$\text{Var}[X] = e^{2\mu + 6^2} (e^{\sigma^2} - 1) = 5 \rightarrow (2)$$

$$\text{from (1)} \Rightarrow \mu + \sigma^2/2 = \ln 30 \quad (3)$$

put (3) in (2):

$$2\mu + \sigma^2 = 2\ln 30$$

$$\Rightarrow e^{2\ln 30} (e^{\sigma^2} - 1) = 5$$

$$\Rightarrow e^{\ln 900} (e^{\sigma^2} - 1) = 5$$

$$\Rightarrow (e^{\sigma^2} - 1) = \frac{5}{900}$$

$$\Rightarrow e^{\sigma^2} = 1 + \frac{5}{900}$$

$$\Rightarrow \sigma^2 = \ln \left(1 + \frac{5}{900} \right)$$

$$\approx 0.0055$$

put in (3) to get

$$\mu = \ln 30 + \ln \left(1 + \frac{5}{900} \right) / 2 \approx 3.4040$$

Q3 Continue

$$Pr(X > 35) = 1 - \Phi \left[\frac{\ln 35 - 3.4040}{\sqrt{0.0055}} \right]$$

$$= 1 - \Phi(2.04) \quad \text{find from table}$$

$$\Rightarrow Pr(X > 35) = 1 - 0.9793 \approx 0.02$$

Q4

$$\textcircled{a} \int_{1/2}^1 \int_0^y (6-6y) dx dy$$

$$= \int_{1/2}^1 (6-6y)y dy$$

$$= \int_{1/2}^1 6y - 6y^2 dy$$

$$= 6 \frac{y^2}{2} - 6 \frac{y^3}{3} \Big|_{1/2}^1 = \frac{1}{2}$$

$$\textcircled{b} f_X(x) = \int_x^1 6(1-y) dy = 6y - 3y^2 \Big|_x^1 = \begin{cases} 3x^2 - 6x + 3, & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \int_0^y 6(1-y) dy = 6 \left[y - \frac{y^2}{2} \right]_0^y = \begin{cases} 6y(1-y), & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Q4 Continue

(c)

$$P_X(x) P_Y(y) \neq P_{X,Y}(x,y)$$

So X, Y are dependent.

Q5

$X = \text{loss amount} \sim \text{uniform}[0, 450]$

density fn
↓

$$Y = \begin{cases} 0 & 0 \leq X < d \\ X-d & d \leq X < 450 \end{cases}$$

$$P_X(x) = \frac{1}{450}$$

$$E[Y] = 120 = \int_d^{450} (x-d) \cdot \frac{1}{450} dx$$

$$\Rightarrow 120 = \left. \frac{(x-d)^2}{2 \cdot 450} \right|_d^{450}$$

Solve for $d \Rightarrow \boxed{d = 121.37}$

$$\boxed{Q6} \textcircled{a} \int_0^1 \int_0^y cxy \, dx \, dy = 1 \quad \leftarrow \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{X,Y}(x,y) \, dx \, dy = 1$$

$$\Rightarrow \int_0^1 \left(cy \frac{x^2}{2} \right) \Big|_0^y \, dy = \int_0^1 c \frac{y^3}{2} \, dy$$

$$= \frac{c}{2} \frac{y^4}{4} \Big|_0^1 = 1 \Rightarrow \boxed{c=8}$$

$$\textcircled{b} \quad P_X(x) = \int_0^x 8xy \, dy = 8x \frac{y^2}{2} \Big|_0^x = 4x^3$$

$$P_{Y|X}(y|x) = \frac{8xy}{4x^3} = \frac{2y}{x^2}$$

$$\textcircled{c} \quad E[Y|X=x]$$

$$= \int_0^x y \cdot P_{Y|X}(y|x) \, dy$$

$$= \int_0^x y \cdot \frac{2y}{x^2} \, dy = \frac{2}{3} x, \quad 0 \leq x \leq 1$$
