King Saud University
College of Science
Department of Mathematics

First Mid-Term Exam, S1, 1446 H.
Math-380 – Stochastic Processes
Time: 2 hours

Answer all questions:

Question 1: [2.5+2.5]

A company has 3 suppliers, designated *A*, *B*, and *C*. The relative amounts of a certain product purchased from each of the suppliers are 50%, 35%, and 15%, respectively. If the proportion of defective produced by each supplier is 1%, 2%, and 3%, respectively.

(a) What is the probability that a randomly selected product is defective?

(b) If a defective random product is selected, find the probability that this product was made by supplier *A*. **Hint:** use Baye's theorem.

Question 2: [5]

Suppose that the price *X* of a particular stock at closing has a log-normal distribution with a mean of $30 and a variance of 5. What is the probability that the price exceeds $35?

Question 3: [2.5+2.5]

(a) Given life expectancy in some country is $\boldsymbol{E}[T] = 70$. Let $G \in \{m, f\}$ denote the gender of that individual (male and female). The statistics show that:

$$\boldsymbol{E}[T|G = m] = 65 \quad \text{and} \quad \boldsymbol{E}[T|G = f] = 72.$$

Find the probability that the gender of that individual is male.

(b) Define the memoryless property, and show how we can use it to find the following: $Pr(X \leq 100|X > 90)$ for the exponential random variable $X\sim\exp(0.04)$.

Question 4: [2.5+2.5]

(a) Assume that the independent identically distributed (i.i.d.) random variables ξ_k , and the random variable N have the following finite moments:

$$\boldsymbol{E}[\xi_k] = \mu, \quad \boldsymbol{Var}[\xi_k] = \sigma^2, \quad \boldsymbol{E}[N] = \upsilon, \quad \boldsymbol{Var}[N] = \tau^2,$$

and consider the random sum $X = \xi_1 + \xi_2 + \cdots + \xi_N$.

1) Derive the formula of $\boldsymbol{E}[X]$.

2) Write the formula of $\boldsymbol{Var}[X]$, (without proof).

(b) An observation is made of a Poisson random variable N with parameter 100. Then, N independent Bernoulli trials are performed, each with probability 0.10 of success. Let Z be the total number of successes observed in the N trials. What are the mean and the variance of Z ?

Question 5: [1+1.5+2.5]

(a) Give the definition of a martingale.

(b) Prove that if $(Z_n)_{n\in\mathbb{N}}$ is a martingale, then $\boldsymbol{E}[Z_n] = \boldsymbol{E}[Z_0]$, for any $n \in \mathbb{N}$.

(c) Let $S_1, S_2, \ldots, S_n, \ldots$ be a sequence of independent random variables such that $\boldsymbol{E}[|S_i|] < \infty$, for all $i = 1, 2, \ldots$. Let $X_0 = 0$, and $X_n = S_1 + S_2 + \cdots + S_n$, $n \geq 1$.

Prove that: $(X_n)_{n\in\mathbb{N}}$ is a martingale if and only if $\boldsymbol{E}[S_n] = 0$, for all $n \geq 1$.

Good Luck

Table 5.1 Area Φ(*x*) Under the Standard Normal Curve to the Left of *X*.

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998