

[Draft]

**KING SAUD UNIVERSITY**  
**COLLEGE OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**

**Semester 471 / MATH-244 (Linear Algebra) / Mid-term Exam 2**

**Max. Marks: 25**

**Max. Time:  $1\frac{1}{2}$  hrs.**

**Note:** Calculators are not allowed.

**Question 1:** [Marks: 3+2]

I. Select the correct choices:

- (i) If  $W = \{(1,1), (1,2)\}$ , then  $W$  is:  
 (a) a vector space (b) a subspace of  $\mathbb{R}^2$  (c) a basis of  $\mathbb{R}^2$  (d) linearly dependent.
- (ii) If  $B = \{u, v, w\}$  is an orthogonal set of non-zero vectors in an inner product space  $E$  and  $\alpha u + \beta v + \gamma w = 0$  for some scalars  $\alpha, \beta$  and  $\gamma$ , then:  
 (a)  $\alpha = \beta = \gamma = 0$  (b)  $u = v = w$  (c)  $\alpha = \gamma, v = 0, u = w$  (d)  $B$  is linearly dependent.
- (iv) If  $A$  is an invertible matrix of order 3, then  $\text{rank}(A)$  is equal to:  
 (a) 0 (b) 1 (c) 2 (d) 3.

II. Give an example for each of the following. You don't have to prove your answers.

- (i) A linearly dependent subset of  $\mathbb{R}^3$  consisting of three vectors in which every two vectors are linearly independent.
- (ii) A matrix having  $\text{rank}$  3 and  $\text{nullity}$  4.

**Question 2:** [Marks: 4 + 3]

Let  $A = \begin{bmatrix} 1 & 1 & 2 & -3 & 5 \\ 2 & 3 & 0 & -1 & -6 \\ 3 & 4 & 2 & -4 & -1 \end{bmatrix}$ . Then:

- (i) Find bases for  $\text{row}(A)$  and  $\text{col}(A)$ .
- (ii) Express the last three columns of the matrix  $A$  as linear combinations of the first two.

**Question 3:** [Marks: 3 + 4]

- (i) Let  $F = \{p = 2 - x, q = 5 + 3x - x^2 + 2x^3, r = 1 + 7x + x^2 + 4x^3\}$ . Then, find the real numbers  $\alpha, \beta, \gamma$  so that  $-3 + 5x^2 + 2x^3 = \alpha p + \beta q + \gamma r$ . Also, show that the set  $F$  is linearly independent in the vector space  $P_3$ .
- (ii) Let  $G = \{v_1, v_2, v_3, v_4, v_5, v_6\}$  be a basis of the vector space  $\mathbb{R}^6$ . Let  $A$  be a  $4 \times 6$  matrix such that  $A(v_1)$  and  $A(v_2)$  are linearly independent vectors in  $\mathbb{R}^4$  and  $A(v_i) = (0, 0, 0, 0)$ , for all  $i = 3, 4, 5, 6$ . Show that the null space  $N(A) = \text{span}(\{v_3, v_4, v_5, v_6\})$ . Also, find  $\text{nullity}(A^T)$ .

**Question 4:** [Marks: 3 + 3]

- (i) Consider a vector space  $E$  of dimension 3. Let  $B = \{u_1, u_2, u_3\}$  and  $C = \{v_1, v_2, v_3\}$  be two ordered bases for  $E$  such that the transition matrix  ${}_C P_B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$  from  $B$  to  $C$ . Compute the coordinate vector  $[v_1 - 2v_2 + 3v_3]_B$ .
- (ii) Consider the matrices  $A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$  in the inner product space  $M_2(\mathbb{R})$  of  $2 \times 2$  real matrices with inner product  $\langle A, B \rangle = \text{trace}(AB^T)$ . Find the angle between the matrices  $A$  and  $B$ . Also, verify the Pythagorean theorem for the same matrices.

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