

[Draft]

**KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS**

Semester 471 / MATH-244 (Linear Algebra) / Mid-term Exam 1

Max. Marks: 25

Max. Time: $1\frac{1}{2}$ hrs.

Note: Calculators are not allowed.

Question 1: [Marks: 5]

Determine whether the following statements are true or false:

- (i) If A and B are diagonal matrices of same size, then $AB = BA$.
- (ii) If A and B are $n \times n$ matrices and A is singular, then AB is also singular.
- (iii) If a matrix A satisfies $A^2 = I_n$, then $A = I_n$ or $A = -I_n$.
- (iv) Any square matrix A satisfies $RREF(A) = I$ iff A is invertible.
- (v) If the linear system $AX = B$ has solutions $u = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$, then $w = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ is another solution of the same system.

Question 2: [Marks: 5 + 5]

- (a) Let the matrix $A = \begin{bmatrix} a & a & a & a \\ b & a & a & a \\ b & b & a & a \\ b & b & b & a \end{bmatrix}$, where $a \neq 0$ and $b \neq a$. Show that $|A| = a(a - b)^3$ and deduce that the matrix A is invertible.

- (b) Let A be the matrix with $A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. Find: $(A^T)^{-1}$, $|2A|$, $adj(A)$, $RREF(A)$.

Also find the solution of the homogeneous linear system $AX = 0$.

Question 3: [Marks: 5 + 5]

- (a) Solve the following system of equations by the Gauss-Jordan elimination method:

$$\begin{aligned} w + x + y + z &= 6 \\ w + y + z &= 4 \\ w + y &= 2. \end{aligned}$$

- (b) Under what condition on α and β , the following system of equations has no non-trivial solution?

$$\begin{aligned} x + 2z &= 0 \\ \alpha x + 8y + 3z &= 0 \\ \beta y + 5z &= 0. \end{aligned}$$

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