

[Draft]

**KING SAUD UNIVERSITY  
COLLEGE OF SCIENCES  
DEPARTMENT OF MATHEMATICS**

**Semester 462 / MATH-244 (Linear Algebra) / Mid-term Exam 1**

**Max. Marks: 25**

**Max. Time:  $1\frac{1}{2}$  hrs.**

**Note:** Scientific calculators are not allowed.

**Question 1:** [Marks: 5]

Determine whether the following statements are true or false and justify your answer:

- (i) If  $A$  and  $B$  are symmetric matrices compatible for the product  $AB$ , then  $AB$  is also symmetric.
- (ii) If the matrix  $A^2$  is invertible, then  $A$  itself is invertible.
- (iii) If the matrix  $\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ -4 & -4 & x \end{bmatrix}$  is its own adjoint, then  $x = 3$ .
- (iv) If matrices  $A$  and  $B$  are compatible for the product  $AB$ , then  $|AB| = |BA|$ .
- (v) If  $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ , then  $(-\frac{1}{11}, \frac{2}{11})$  is a solution of the equation  $A^{-1} = xA + yI_2$ .

**Question 2:** [Marks: 4 + 3 + 3]

- (a) Find the matrix  $A$  if  $A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & -2 \end{bmatrix}$ .
- (b) Show that  $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ -(b+c-a) & -(c+a-b) & -(a+b-c) \end{vmatrix} = 0$ .
- (c) Let  $A$  be a square matrix of size  $n$  with  $|A| = 3$  and  $|adj(A)| = 27$ . Find  $n$ .

**Question 3:** [Marks: 3 + 3 + 4]

- (a) Solve the matrix equation:  $XA = B$ , where  $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ -5 & 0 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} -2 & 1 & 1 \\ 8 & 1 & -5 \\ 4 & 3 & -3 \end{bmatrix}$ .
- (b) Solve the following system of linear equations:
$$\begin{aligned} x + 3y + 3z &= 1 \\ x + 3y + 5z &= -1 \\ x + 2y - z &= 2. \end{aligned}$$
- (c) Find the value of  $m$  for which the following system of linear equations admits a unique solution and then find this uniquely existing solution.
$$\begin{aligned} x + y + z &= 1 \\ x + y + 2z &= 0 \\ 2x - y - z &= -1 \\ x - 2y + z &= m. \end{aligned}$$

\*\*\*!