151 Discrete Mathematics
K. Rosen, Discrete Mathematics and Its Applications, 7th edition

Chapter 1 The Foundations: Logic and Proofs

1.1. Propositional Logic

Ex.1, Ex.2, Def.1, Ex.3, Ex.4, Def.2, Ex.5, Def.3, Ex.6, Def.5, Ex.7, converse, contrapositive, inverse, Def.6, Ex.10, Ex.11, precedence of logical operators.

1.3. Propositional Equivalences

Def.1, Ex.1, Def.2, Ex.2, Ex.3, Ex.4, Table 6, Table 7, Table 8, Ex.5, Ex.6, Ex.7, Ex.8.

1.4. Predicates and Quantifiers

Ex.1, Ex.2, Ex.3, Ex.4, Ex.5, $P(x_1, x_2, \cdots, x_n)$, Def.1, Ex.8, Ex.9, Ex.10, Ex.11, Ex.13, Def.2, Ex.14, Ex.15, Ex.16, Ex.17, negating expressions, Table 2, Ex.21, Ex.22.

1.7. Introduction to Proofs

Def.1, Ex.1, Ex.2, Ex.3, Ex.4, Ex.5, Ex.6, Def.2, Ex.7, Ex.8, Ex.10, Ex.11, Ex.12, Ex.13, Ex.14, Ex.15, Ex.18.

1.8. Proof Methods and Strategy

Ex.1, Ex.3, Ex.4, Ex.6, Ex.7, Ex.9, Ex.10, Ex.13, Ex.17.

Chapter 5 Induction and Recursion

5.1. Mathematical Induction

Ex.1, Ex.2, Ex.3, Ex.4, Ex.5, Ex.6, Ex.8

5.2. Strong Induction and Well-Ordering

Ex.2.

Q.1. Let $\{a_n\}$ be a sequence defined inductively as:

$a_1 = 1$, $a_2 = 2$, $a_{n+1} = 2a_n + a_{n-1}, \forall\, n \geq 2$. Prove that: $a_n \leq \left(\frac{5}{2}\right)^n$, for all $n \geq 2$.

Q.2. Let $\{a_n\}$ be *Fibonacci sequence,* which is defined inductively as:

$a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}\ \forall\, n \geq 3$. Prove that: $a_n \leq \left(\frac{1+\sqrt{5}}{2}\right)^n$, for all $n \geq 1$.

Q.3. Let $\{a_n\}$ be a sequence defined inductively as:

$a_0 = 1$, $a_1 = 1$, $a_n = 2a_{n-1} + a_{n-2}, \forall\, n \geq 2$. Prove that: a_n *is odd,* for all $n \geq 0$.

Chapter 9: Relations

9.1. Relations and Their Properties

Ex.2, Ex.3, Def.2, Ex.4, Ex.5, Def.3, Ex.7, Ex.8, Ex.9, Def.4, Ex.10, Ex.11, Ex.12, Def.5, Ex.13, Ex.14, Ex.15, Ex.17, Ex.18, Ex.19, Def.6, Ex.20, Ex.21, Def.7, Ex.22, Th.1-no proof.
(Definition of inverse relation and complementary relation, Exercise 27).

9.3. Representing Relations

Ex.1, Ex.2, Ex.3, Ex.4, Ex.5, Ex.6, Def.1, Ex.7, Ex.8, Ex.9, Ex.10.

9.5. Equivalence Relations

Def.1, Def.2, Ex.1, Ex.2, Ex.3, Ex.6, Ex.7, Def.3, Ex.8, Ex.9, Th.1 (no proof), Ex.12, Th.2 (no proof), Ex.13, Ex.14.

9.6. Partial Orderings

Def.1, Ex.1, Ex.2, Ex.3, Ex.4, Def.2, Ex.5, Def.3, Ex.6, Ex.7, Hasse diagrams, Ex.12&13.

Chapters 10: Graphs

10.1: Graphs and Graph Models

All until graph models (which is not included).

10.2: Graph Terminology and special Types of Graphs

Def.1, Def.2, Def.3, Ex.1, Th.1, Ex.3, Th.2 (no proof), Def.4, **Definition** of (**n-regular**) before exercise 53 page 667, **Example**: Find the regularity type of the following graphs:
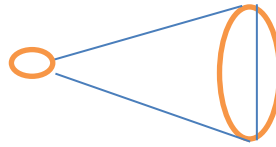

Figure 1

Figure 2

Theorem: If G=(V,E) is an r-regular graph with $|V|=n$, then $|E|=\frac{n.r}{2}$.

Apply the previous theorem on Example E1(a,b), Ex.5 (Complete graph) p.655, Ex. 6,

Theorem: If $Kn=(V,E)$, then $|E|=\frac{n(n-1)}{2}$.

Def.6(Bipartite graph) p. 656, Ex.9, Ex.10, Ex.11, Ex.13(Complete bipartite graph) p.658.

Theorem: If $Km,n = (V_1 \cup V_2, E)$ such that $|V_1| = m$ and $|V_2| = n$, then $|E| = mn$.

10.3: Representing Graphs and Graph Isomorphism

Ex.1, Def.1, Ex.3, Ex.4, Ex.6, Ex.8, Ex.9, Ex.10.

10.4: Connectivity

Def.1, Ex.1, Def.2, Def.3, Ex.4, Th.1 (no proof), Ex.5.

Chapters 11: Trees

11.1: Introduction to Trees

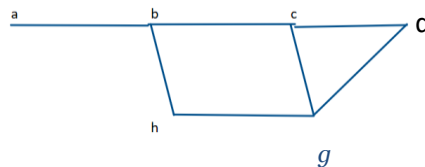Def. 1, Ex.1, Th.1(no proof), Def. 2, Ex.2, Def.3, Ex.4, Th.2(no proof).

11.2: Applications of Trees

Ex.1.

11.4: Spanning Trees

Def.1, Ex.1, depth-first search tree (with choosing a specific root), Ex.3, Ex.4, Exercise 13 p.795, breadth-first search tree (with choosing a specific root), Ex. 5, **Example**: For the graph G,

(a) Find (breadth-first search tree) with root g   (b) Find (depth-first search tree) with root g.

Chapter 12: Boolean Algebra

12.1: Boolean Functions

Ex.1, Ex.4, Ex.5, Table 5, Ex.10, Ex.11, Ex.12.

12.2: Representing Boolean Functions

Look at the attached file for this section.

12.4: Minimization of Circuits

Ex.1, Ex.2, Ex.3, Ex.4 and all definitions up to page. 835.

Ex 1, Def 1

Definition: The sum-of-products expansion CSP(f) is the sum of minterms that represent the function, i.e. the minterms that has the value 1.

Ex 2, Ex 3,

Definition: A *maxterm* of the Boolean variables $x_1 x_2, \cdots x_n$ is a Boolean sum $y_1 + y_2 + \cdots + y_n$, where $y_i = x_i$ or $y_i = \bar{x}_i$. Hence, a maxterm is a sum of n literals, with one literal for each variable.

Definition: The product-of-sum expansion CPS(f) is the product of maxterms that has the value 0.

Example: Find the CPS(f) of the function in example 3.

1- Using tables:
CPS(f)=$(\bar{x} + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(x + \bar{y} + \bar{z})(x + y + \bar{z})(x + y + z)$

2- Using Boolean identities:
$$CPS(f) = [CSP(f^d)]^d$$
$$* \ f^d(x,y,z) = xy + \bar{z}$$
$$* \ CSP(f^d) = xy(z + \bar{z}) + (x + \bar{x})(y + \bar{y})\bar{z}$$
$$= xyz + xy\bar{z} + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$$
$$= xyz + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$$
$$*[CSP(f^d)]^d = (x + y + z)(x + y + \bar{z})(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + \bar{z})$$

Thus, $CPS(f) = (x + y + z)(x + y + \bar{z})(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + \bar{z})$

Example:
Find CSP(f) and CPS(f) where $f(x,y) = \bar{x} + y$.
The table for $f(x,y)$ is:

x	y	$\bar{x}$	$f(x,y) = \bar{x} + y$
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

Thus, from table we have:
1- CSP(f)=$xy + \bar{x}y + \bar{x}\bar{y}$

2- CPS(f)= $(\bar{x} + y)$

Using Boolean identities:

1- CSP(f) $= \bar{x}(y + \bar{y}) + (x + \bar{x})y$
$$= \bar{x}y + \bar{x}\bar{y} + xy + \bar{x}y$$
$$= \bar{x}y + \bar{x}\bar{y} + xy$$

2- $CPS(f) = [CSP(f^d)]^d$
$$= [CSP(\bar{x}y)]^d$$
$$= [\bar{x}y]^d$$
$$= \bar{x} + y$$

Exercises: 1-2(a,b)-3(a,b)-11(a,b).