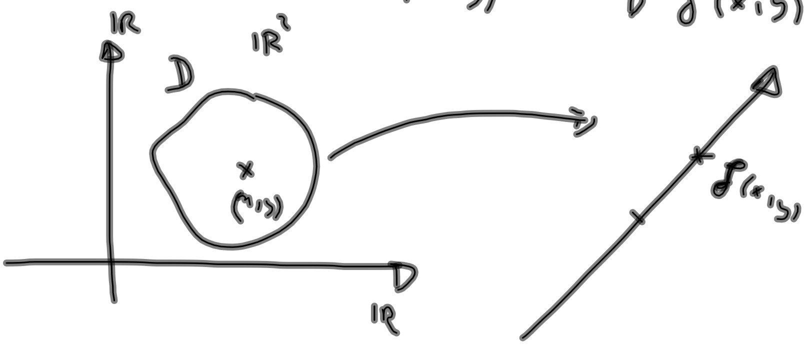
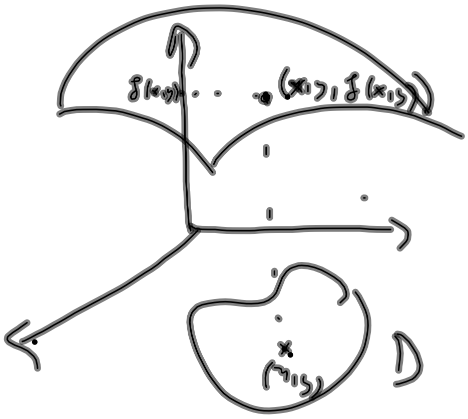
Functions of two variables

Definition: $f : D \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$

$(x,y) \longrightarrow f(x,y)$



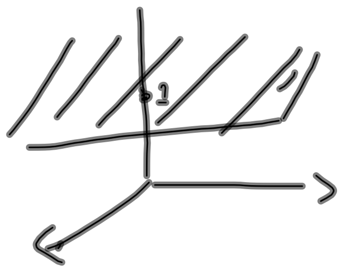
The graph of f is a surface in the space



Examples:

1. $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$: Constant function

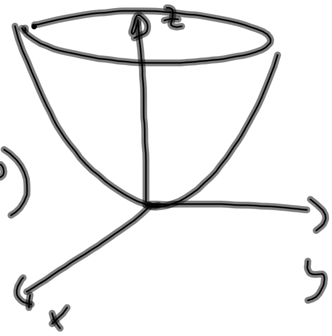
   $(x,y) \longmapsto 1$



2. $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$

   $(x,y) \longmapsto x^2 + y^2$

parabolic surface ([illegible])



Domain of definition of f:

$$D_f = \{ (x,y) \in \mathbb{R}^2 : \ f(x,y) \ \underline{\text{exists}} \}$$

Examples: (1) $f(x,y) = x^3 + y^2 + 3xy + 1$

$$D_f = \mathbb{R}^2$$

Any polynomial function has $D_f = \mathbb{R}^2$

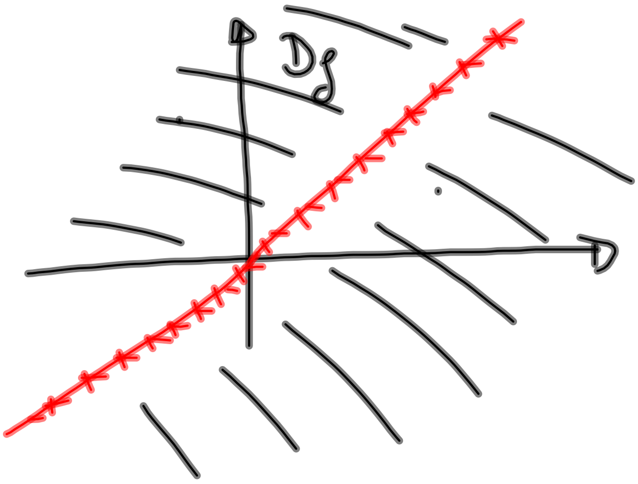
(2) $f(x,y) = \dfrac{1}{x^2+y^2}$ $\quad D_f = \mathbb{R}^2 \setminus \{(0,0)\}$

(3) $f(x,y) = \dfrac{x^3+y^2}{x-y}$

$$D_f = \left\{ (x,y) \in \mathbb{R}^2 : \ \frac{x^3+y^2}{x-y} \text{ exists} \right\}$$

$$= \{ (x,y) \in \mathbb{R}^2 : \ x - y \neq 0 \}$$

$$\boxed{\begin{aligned} x - y &= 0 \\ x &= y \end{aligned}}$$



(4) $\dfrac{x^3 - y^2}{|x| - |y|}$