Name: **ID:** **Section:** 54030 **Mark:**

King Saud University

College of Sciences, Department of Mathematics

1444/Semester-3/ MATH 380/ Quiz-1

Marks: 10 **Max. Time: 35 Minutes**

Answer the following questions.

Q1: [2.5+2.5]

(a) Prove that if $T \sim \exp(\lambda)$ then $\Pr(T > t+s | T > s) = \Pr(T > t)\ \forall t, s \geq 0$

(b) The lifetime T of a certain component has an exponential distribution with parameter $\lambda = 0.03$. Find $\Pr(T \leq 130 | T > 100)$.

Q2: [2.5+2.5]

(a) Twelve independent random variables, each uniformly distributed over the interval (0, 1], are added, and 6 is subtracted from the total. Determine the mean and variance of the resulting random variable.

(b) Given independent exponentially distributed random variables S and T with common parameter λ, determine the probability density function of the sum $R = S + T$ and identify its type by name.

The Model Answer

Q1: [2.5+2.5]

(a)

$$\Pr(T > t+s \mid T > s) = \frac{\Pr(T > t+s, T > s)}{\Pr(T > s)}$$

$$= \frac{\Pr(T > t+s)}{\Pr(T > s)}$$

$\because T \sim \exp(\lambda)$

$$\therefore \Pr(T > t+s \mid T > s) = \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}} = e^{-\lambda t}$$

$$= R(t) = \Pr(T > t)$$

(b)

$$\Pr(T \le 130 \mid T > 100) = 1 - \Pr(T > 130 \mid T > 100)$$

$$= 1 - \Pr(T > 30)$$

$$= 1 - e^{-0.03(30)}$$

$$\therefore \Pr(T \le 130 \mid T > 100) = 1 - e^{-0.9} \simeq 0.59$$

Q2: [2.5+2.5]

(a)

$\because X_k \sim$ uniform $(0,1),\ k = 1, 2, \ldots, 12$

$\therefore E(X_k) = \frac{1}{2}(a+b),\ Var(X_k) = \frac{1}{12}(b-a)^2$

$\therefore E(X_k) = \frac{1}{2}(0+1) = \frac{1}{2},\ Var(X_k) = \frac{1}{12}(1-0)^2 = \frac{1}{12}$

For $Z = X_1 + X_2 + \ldots + X_{12} - 6$

$E(Z) = 12\left(\frac{1}{2}\right) - 6 = 0,\ E(6) = 6$

$Var(Z) = 12\left(\frac{1}{12}\right) - 0 = 1,\ Var(6) = 0$

(b)

$\because S, T \sim \exp(\lambda),\ R = S + T$

$\therefore R \sim \text{Gamma}(2, \lambda)$

$$\therefore f_R(r) = \frac{\lambda^2}{\Gamma(2)} r^{2-1} e^{-\lambda r},\ r \ge 0$$

$$\therefore f_R(r) = \frac{\lambda^2}{1!} r e^{-\lambda r}$$

$$\therefore f_R(r) = \lambda^2 r e^{-\lambda r},\ r \ge 0$$

which is the Gamma probability density function.
