Name: **ID:** **Section:** **Mark:**

King Saud University

College of Sciences, Department of Mathematics

1444/Semester-2/ MATH 380/ Quiz-1

Marks: 10 **Max. Time: 35 Minutes**

Answer the following questions.

Q1: [2.5+2.5]

(a) A fraction $p = 0.04$ of the items coming off a production process are defective. If a random sample of 9 items is taken from the output of the process, what is the probability that the sample contains exactly one defective item? What is the probability that the sample contains one or fewer defective items?

(b) Given independent exponentially distributed random variables S and T with common parameter λ, determine the probability density function of the sum $R = S + T$ and identify its type by name.

Q2: [2.5+2.5]

(a)The lifetime T of a certain component has an exponential distribution with parameter $\lambda = 0.03$. Find $\Pr(T \leq 130 | T > 100)$.

(b) Suppose that X is a Poisson distributed random variable with mean $\lambda = 3$. Determine $\Pr\{X \leq \lambda\}$.

The Model Answer

Q1: [2.5+2.5]

(a) If $X \sim Bin(n,p)$ then $\Pr\{X=x\} = \binom{n}{x} p^x q^{n-x},\ 0<p<1,\ q=1-p$

(i)

$$\Pr\{X=1\} = \binom{9}{1}\ 0.04^1\ 0.96^8 \simeq 0.25970$$

(ii)

$$\begin{aligned}\Pr\{X \le 1\} &= \Pr\{X=0\} + \Pr\{X=1\} \\ &= \binom{9}{0}\ 0.04^0\ 0.96^9 + \binom{9}{1}\ 0.04^1\ 0.96^8 \\ &\simeq 0.95223\end{aligned}$$

(b)

$\because S, T \sim \exp(\lambda)$

$\therefore R \sim \text{Gamma}(2,\lambda)$ where $R = S + T$ and the pdf is given by

$$f_R(r) = \frac{\lambda^2}{\Gamma(2)} r^{2-1} e^{-\lambda r},\ r \ge 0$$

$$\therefore f_R(r) = \frac{\lambda^2}{1!} r e^{-\lambda r}$$

$$\therefore f_R(r) = \lambda^2 r e^{-\lambda r},\ r \ge 0$$

which is the Gamma probability density function.

Q2: [2.5+2.5]

(a)

$$\begin{aligned}\Pr(T \le 130 | T > 100) = 1 - \Pr(T > 130 | T > 100) &= 1 - \Pr(T > 30) \\ &= 1 - e^{-0.03(30)} \simeq 0.59\end{aligned}$$

(b)

$$\Pr\{X \le 3\} = \sum_{x=0}^{3} \frac{e^{-3} 3^x}{x!}$$

$$= e^{-3}\left[1 + \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right]$$

$$= \ 13\ e^{-3} \simeq 0.6472$$