

SECOND MID TERM EXAMINATION, SEM. II, 2025
DEPT. MATH., COLLEGE OF SCIENCE
KING SAUD UNIVERSITY
MATH: 107 FULL MARK: 25 TIME: 90 MIN.

Q1. [2+2+2=6]

- (a) A constant force $\mathbf{F} = \langle 5, -3, 1 \rangle$ moves a body from point $P(1, 1, 1)$ to point $Q(9, 4, 7)$ along a straight line. Find the work done.
- (b) Consider the vectors $\mathbf{a} = \langle x, 0, 0 \rangle$, $\mathbf{b} = \langle 0, y, 0 \rangle$ and $\mathbf{c} = \langle 0, 0, z \rangle$ with $\mathbf{a} + 2\mathbf{b} + 3\mathbf{c} = \langle 1, 2, 3 \rangle$. Find values of x , y and z .
- (c) Show that the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are mutually orthogonal.

Q2. [3+2+2+2=9]

- (a) Find an equation of the plane through the points $P(1, -2, 0)$, $Q(2, 0, 3)$ and $R(0, -2, -3)$.
- (b) Let p_1 and p_2 be two planes defined by their equations:

$$p_1 : x - 2y + 2z = 3$$

$$p_2 : 2x + y - z = 1$$

- (i) Prove that p_1 and p_2 are not parallel.
- (ii) Find parametric equations of the line of intersection of the planes p_1 and p_2 .
- (iii) Find the distance between the point $A(1, -1, 3)$ and the plane p_1 .

Q3. [3+4+3=10]

- (a) Let $\mathbf{r}(t) = \ln(1 - t)\mathbf{i} + \sin t\mathbf{j} + t^2\mathbf{k}$. Find the domain of $\mathbf{r}$. Also, find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.
- (b) If $\mathbf{r}(t) = e^t(\cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k})$ is the position vector of a moving point P , find its velocity, acceleration, and speed at $t = \frac{\pi}{2}$.
- (c) Sketch the graph of the surface $9(x^2 + z^2) + 4y^2 = 36$ in an xyz coordinate system, describe the traces on the coordinate system, and identify the surface.

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- Q1. [2+2+2=6]** (a) The displacement vector $PQ = \langle 8, 3, 6 \rangle$. Hence, work done $= \mathbf{F} \cdot \mathbf{PQ} = \langle 5, -3, 1 \rangle \cdot \langle 8, 3, 6 \rangle = 37$ units.
 (b) $\langle 1, 2, 3 \rangle = a + 2b + 3c = \langle x, 0, 0 \rangle + 2\langle 0, y, 0 \rangle + 3\langle 0, 0, z \rangle = \langle x, 2y, 3z \rangle \Rightarrow x = y = z = 1$.
 (c) $a \cdot b = \langle 1, 0, 0 \rangle \cdot \langle 0, 1, 0 \rangle = 0$. Similarly, $b \cdot c = 0$ and $c \cdot a = 0$.

- Q2. [3+2+2+2=9]** (a) Let $a = PQ = \langle 1, 2, 3 \rangle$ and $b = PR = \langle -1, 0, -3 \rangle$. Then $a \times b = \langle -6, 0, 2 \rangle$. So the plane contains $P(1, -2, 0)$ and has normal vector $a \times b = \langle -6, 0, 2 \rangle$. This plane, then, has equation $-6(x - 1) + 2(z - 0) = 0$, that is, $-6x + 2z + 6 = 0$.
 (b) (i) We have normal vectors $n_1 = \langle 1, -2, 2 \rangle$ and $n_2 = \langle 2, 1, -1 \rangle$ show that $\frac{1}{2} \neq \frac{-2}{1}$, then $p_1 \nparallel p_2$.
 (ii) Considering the augmented matrix, we have

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 2 & 1 & -1 & 1 \end{bmatrix} \\ \sim \begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

where, we have $x = 3 + 2y - 2z$ and $y = -1 + z$. Using $z = t$, we get $x = 1$, $y = -1 + t$ and $z = t$, for all $t \in \mathbb{R}$.

- (iii) The distance from the point $A(1, -1, 3)$ to the plane is $d = \frac{|1 - 2(-1) + 2(3) - 3|}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{6}{\sqrt{9}} = 2 \Rightarrow d = 2$.

- Q3. [3+4+3=10]** (a) Domain of r is $(-\infty, 1]$. $r'(t) = \frac{-1}{1-t}i + \cos t j + 2tk$. $r''(t) = \frac{-1}{(1-t)^2}i - \sin t j + 2k$.
 (b) $r(t) = e^t \cos t i + e^t \sin t j + e^t k$
 $v(t) = r'(t) = (e^t \cos t - e^t \sin t)i + (e^t \cos t + e^t \sin t)j + e^t k$
 $a(t) = r''(t) = -2e^t \sin t i + 2e^t \cos t j + e^t k$.
 $v(\frac{\pi}{2}) = -e^{\frac{\pi}{2}}i + e^{\frac{\pi}{2}}j + e^{\frac{\pi}{2}}k = e^{\frac{\pi}{2}}(-i + j + k)$
 $a(\frac{\pi}{2}) = -2e^{\frac{\pi}{2}}i + e^{\frac{\pi}{2}}k = e^{\frac{\pi}{2}}(-2i + k)$.
 $\text{Speed} = \sqrt{e^{\pi} + e^{\pi} + e^{\pi}} = \sqrt{3e^{\pi}} = \sqrt{3}e^{\frac{\pi}{2}}$.
 (c) The given equation of the surface can be written as $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 1$, which is an ellipsoid.
 xy -trace is $\frac{x^2}{4} + \frac{y^2}{9} = 1$, which is an ellipse; yz -trace is $\frac{y^2}{9} + \frac{z^2}{4} = 1$, which is an ellipse; xz -trace is $\frac{x^2}{4} + \frac{z^2}{4} = 1$, which is a circle. Sketch is given at the last page.

