# FINAL EXAMINATION, SEMESTER I, 2024
## Dept. Math., College of Science, KSU
### Math: 107 Full Mark: 40 Time: 3 Hours

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**Q1**.[3+2+4=9]

(a) Solve the system of linear equations by Gaussian elimination:

$$x + 2y - z = 2$$
$$x - 2y + 3z = 1$$
$$x + 2y - z = 2$$

(b) Explain why the above system of linear equations cannot be solved by Cramer's rule.

(c) Find the inverse of the matrix $A$ by using $adj(A)$, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

**Q2**. [3+3=6]

(a) Find the volume of a box having adjacent sides $AB$, $AC$ and $AD$ where $A(2, 1, -1)$, $B(3, 0, 2)$, $C(4, -2, 1)$ and $D(5, -3, 0)$.

(b) Find the velocity, speed and acceleration of a particle that moves along the plane curve

$$\mathbf{r}(t) = \langle t\sin(2t); t\cos(2t)\rangle \text{ at } t = 0.$$

**Q3**. [3+2+3=8]

(a) The position vector of a moving point at time $t$ is $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$. Find the tangential and normal components of acceleration, and curvature.

(b) Find the domain of the function defined by $f(x, y) = \sqrt{1 + y - x} + \sqrt{x + y - 1} + \ln(1 - x^2 - y)$.

(c) Prove that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^4+y^3}{y^2+(y^2+x^2)^2}$$

does not exit.

**Q4**. [3+2+3=8]

(a) Use Chain rule to find $\frac{\partial p}{\partial r}$ and $\frac{\partial p}{\partial s}$ if $p = u^2 + 3v^2 - 4w^2$ with $u = 2r - s$, $v = -r + 2s$, $w = r + s$.

(b) If $w = f(x^2 + y^2)$, show that $y\frac{\partial w}{\partial x} - x\frac{\partial w}{\partial y} = 0$.

(c) Let $f(x, y, z) = xy^2e^z$. Find the directional derivative of $f$ at the point $P(2, -1, 0)$ in the direction of vector $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

**Q5** [3+4+2=9]

(a) Identify the surface $z = x^2 + y^2$, and find an equation for the tangent plane and parametric equations for the normal line to the given surface at the point $(-1, 1, 2)$.

(b) Find the local extrema and saddle points of the function defined by

$$f(x, y) = 9x^2y - 6x^3 + y^3 - 12y.$$

(c) Find the three positive numbers whose sum is 1 and whose product is a maximum.