Differential and Integral Calculus (MATH-205)

MT Exam/Semester I (2022-23)   Time Allowed: 120 Minutes

Date: Monday, October 10, 2022   **Maximum Marks:** 30

Note: Attempt all SIX questions and give detailed solutions. Read statements of the questions carefully and make sure you have answered each question completely.

Question 1: $(4°)$ Determine whether the following sequence converges or diverges. Find its limiting value as $n \to \infty$.

$$\left\{ \left(1 + \frac{7}{8n^3} \right)^{n^3} \right\}_{n=1}^{\infty}.$$

Question 2: $(5°)$ Determine whether the series $\sum\limits_{n=1}^{\infty} \ln \left(\frac{n}{n+1} \right)$ converges or diverges. Find its sum, if it converges.

Question 3: $(3°)$ Determine whether the infinite series $\sum\limits_{n=1}^{\infty} n \tan \frac{1}{n}$ converges or diverges. Is it a positive term series?

Question 4: $(6°)$ Find the power series representation of $f(x) = \frac{1+x}{(1-x)^2}$. Find interval of convergence of this series. Hence, find the sum of the series $\sum\limits_{n=0}^{\infty} \frac{2n+1}{2^n}$.

Question 5: $(6°)$ Approximate $\int_0^{\frac{1}{2}} x^2 \cos x^3 \, dx$ using first four non-zero terms of the Maclaurin series. Find the exact value of the definite integral and the absolute error. Use 5 decimal point accuracy in your working.

Question 6: $(6°)$ Given points $A(4, 2, 3)$, $B(8, 1, 8)$, $C(6, 4, 7)$, and $D(12, 5, 5)$. Find (i) the angle (in degrees) between $\overrightarrow{AB}$ and $\overrightarrow{CD}$, (ii) the component of $\overrightarrow{CD}$ along $\overrightarrow{AB}$ and vice versa, (iii) a vector of magnitude $\sqrt{2}$ in the direction of $\overrightarrow{AC}$, and (iv) find k if the magnitude of $\overrightarrow{AE}$ is $\sqrt{17}$, where $E(k, -1, \frac{1}{3})$.

—- Good Luck —-