Differential and Integral Calculus (MATH-205)

MT-I Exam/Fall 2023 Time Allowed: 1.5 Hours

Date: Wednesday, October 4, 2023 Maximum Marks: 25

Note: Attempt all FIVE questions and give detailed solutions. Read statements of the questions carefully and make sure you have answered each question completely.

Question 1: (4°) Find the *n*th term of the sequence $-\frac{1}{2}$, $\frac{4}{5}$, $-\frac{9}{10}$, $\frac{16}{17}$, $-\frac{25}{26}$, \cdots . Determine whether it converges or diverges, and if it converges, find its limit.

Question 2: (5°) Determine the *n*th partial sum of the series $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$. Hence, determine whether it converges or diverges. Find its sum, if it converges.

Question 3: (4°) Determine whether the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{1+n^2}$ converges or diverges.

Question 4: (6°) Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{\ln(n+1)}$ absolutely convergent, conditionally convergent, or divergent.

Question 5: (6°) Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2x-1)^n}{n \, 10^n}$.