

Solution of the first examination Math 106 Semester I
October 2025

Question 1 : (2+3+3)

1. $F'(x) = 2x\sqrt{1+x^{12}} - 2\sqrt{1+16x^6}. \quad (1)+(1)$

2.

$$\begin{aligned} \int \frac{dx}{x \ln x \ln(\ln x)} &\stackrel{u=\ln x}{=} \int \frac{du}{u \ln u} \quad (1) \\ &\stackrel{t=\ln u}{=} \int \frac{dt}{t} = \ln |t| + c \quad (1) \\ &= \ln |\ln(\ln x)| + c. \quad (1) \end{aligned}$$

3.

k	x_k	$f(x_k)$	m_k	$m_k f(x_k)$
0	0	1	1	1
1	1	$\frac{1}{2}$	2	1
2	2	$\frac{1}{3}$	2	$\frac{2}{3}$
3	3	$\frac{1}{4}$	2	$\frac{1}{2}$
4	4	$\frac{1}{5}$	2	$\frac{2}{5}$
5	5	$\frac{1}{6}$	1	$\frac{1}{6}$
				3.7333

(2.5)

$$\int_0^5 \frac{dx}{1+x} \approx 1.8666. \quad (0.5)$$

Question 2 : (3+2+3)

1. $-3 \int_3^5 \sqrt{2x-6} dx = -8$, then $c = \frac{35}{9}. \quad (2) + (1)$

2. If $y = \sqrt{(x^2+1)^3 \sqrt{5x+4}}$, $\ln y = \frac{3}{2} \ln(x^2+1) + \frac{1}{4} \ln(5x+4)$,
(0.5)

$$y' = y \left(\frac{3x}{x^2+1} + \frac{5}{4(5x+4)} \right). \quad (1.5)$$

$$3. \int \frac{x}{\sqrt{(x-1)(x+1)}} dx = \sqrt{x^2-1} + c. \quad (1.5)$$

$$\int \frac{\cosh^{-1} x}{\sqrt{(x-1)(x+1)}} dx \stackrel{u=\cosh^{-1} x}{=} \int u du = \frac{1}{2} (\cosh^{-1} x)^2 + c. \quad (1.5)$$

Question 3 : (3+3+3)

1.

$$\begin{aligned} \int \frac{x dx}{\sqrt{(1-4x^4)}} dx &\stackrel{2x^2=t}{=} \frac{1}{4} \int \frac{dt}{\sqrt{1-t^2}} \quad (1.5) \\ &= \frac{1}{4} \sin^{-1}(2x^2) + c. \quad (1.5) \end{aligned}$$

2.

$$\begin{aligned} \int \frac{dx}{x\sqrt{x^8-16}} &\stackrel{4u=x^4}{=} \frac{1}{16} \int \frac{du}{u\sqrt{u^2-1}} \quad (1.5) \\ &= \frac{1}{16} \sec^{-1}\left(\frac{x^4}{4}\right) + c. \quad (1.5) \end{aligned}$$

3.

$$\begin{aligned} \int \frac{dx}{\sec x \sqrt{9 \sin^2 x - \sin^4 x}} &= \int \frac{\cos x dx}{\sin x \sqrt{9 - \sin^2 x}} \\ &\stackrel{3u=\sin x}{=} \frac{1}{3} \int \frac{du}{u\sqrt{1-u^2}} \quad (1.5) \\ &= -\frac{1}{3} \operatorname{sech}^{-1}\left(\frac{\sin x}{3}\right) + c. \quad (1.5) \end{aligned}$$