KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Semester 462 / MATH-244 (Linear Algebra) / Mid-term Exam 2

Max. Marks: 25 **Max. Time:** $1\frac{1}{2}$ **hrs.**

Note: Calculators are not allowed.

Question 1: [Marks: 1+1+1+1+1**]**

Which of the given choices are correct?

(i) Let $\boldsymbol{P}_2$ be the vector space of all real polynomials in one variable of degree ≤ 2 and S = $\{u, v\} \subseteq \boldsymbol{P}_2$. If $\boldsymbol{E}$ denotes the set of all linear combinations of the vectors in $\boldsymbol{S}$, then the set $\boldsymbol{E}$ is equal to:

(a) $\{\alpha u + v \mid \alpha \in \mathbb{R}\}$ (b) $\{u + \beta v \mid \beta \in \mathbb{R}\}$ (c) $\boldsymbol{P}_2$ (d) a vector space.

(ii) If $\boldsymbol{W} = \{w_1, w_2, w_3, w_4\}$ spans the vector space $\boldsymbol{V}$, then:

(a) $dim(\boldsymbol{V}) = 3$ (b) $dim(\boldsymbol{V}) = 4$ (c) $dim(\boldsymbol{V}) > 4$ (d) $dim(\boldsymbol{V}) \leq 4$.

(iii) Consider the vector space $\mathbb{R}^2$ with ordered basis $\boldsymbol{B} = \{(1,0), (1,2)\}$. If $v \in \mathbb{R}^2$ with the coordinate vector $[\boldsymbol{v}]_{\boldsymbol{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then:

(a) $v = (1,0)$ (b) $v = (3,4)$ (c) $v = (1,2)$ (d) $v = (2,2)$.

(iv) Which of the following matrices cannot be a transition matrix?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$.

(v) If $\boldsymbol{A}$ is an invertible matrix of order 3, then $rank(\boldsymbol{A})$ is equal to:

(a) 0 (b) 1 (c) 2 (d) 3.

Question 2: [Marks: 3 + 2 + 2 + 3]

Consider the matrix $\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 2 \end{bmatrix}$.

(a) Find a basis $\boldsymbol{B_1}$ for the null space $\boldsymbol{N}(\boldsymbol{A})$.
(b) Find a basis $\boldsymbol{B_2}$ for the column space $col(\boldsymbol{A})$.
(c) Find *nullity* and *rank* of the matrix $\boldsymbol{A}$.
(d) Show that $\boldsymbol{B_1} \cup \boldsymbol{B_2}$ is a basis for the Euclidean space $\mathbb{R}^3$.

Question 3: [Marks: 3 + 7]

Consider a vector space $\boldsymbol{E}$ of dimension 3. Let $\boldsymbol{B} = \{u_1, u_2, u_3\}$ and $\boldsymbol{C} = \{v_1, v_2, v_3\}$ be two ordered bases for $\boldsymbol{E}$ such that the transition matrix ${}_{\boldsymbol{C}}\boldsymbol{P}_{\boldsymbol{B}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ from $\boldsymbol{B}$ to $\boldsymbol{C}$. Then, compute:

(a) Transition matrix ${}_{\boldsymbol{B}}\boldsymbol{P}_{\boldsymbol{C}}$ from $\boldsymbol{C}$ to $\boldsymbol{B}$.
(b) Coordinate vectors $[v_1 - v_2]_{\boldsymbol{C}}$, $[v_1 - v_2]_{\boldsymbol{B}}$ and $[v]_{\boldsymbol{C}}$, where $v = v_1 - 2u_2 + v_3$.

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