King Saud University
College of Sciences
Department of Mathematics
Math-244 (Linear Algebra); Mid-term Exam; Semester 2 (1442)

Max. Marks: 30 **Time: 2 hours**

<u>**Note**</u>: ***Attempt all the five questions***!

Question 1: [Marks: 3+3]

a) **Let A** = $\begin{bmatrix} -1 & 1 & 3 & 0 \\ 1 & 2 & 3 & -2 \\ 0 & -1 & -2 & 7 \\ 2 & 1 & 0 & 6 \end{bmatrix}$. Then:

i) **Find** the **reduced row echelon form** of the **matrix A**.

ii) <u>**Use** the reduced row echelon form to</u> **show** that the **matrix A** is **not invertible**.

b) **Let** $\mathbf{X} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$. **Find** the value of λ such that $\mathbf{X}^8 - 4\lambda\mathbf{I} = \mathbf{O}$.

Question 2: [Marks: 3+3]

a) **Let X** = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. **Find** the **matrix Y** such that $(\mathbf{2X} + \mathbf{Y})^{-1} = \boldsymbol{adj}(\mathbf{X})$.

b) **Let A** = $\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$ and **B** = $\begin{bmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{bmatrix}$. **Show** that $\boldsymbol{det}(\mathbf{B}) = -\boldsymbol{det}(\mathbf{A})$.

Question 3: [Marks: 3+3].

a) **Find** the value/s of α such that the following **linear system**

$$\begin{aligned} \boldsymbol{x} + \boldsymbol{y} + \tfrac{\alpha}{3}\boldsymbol{z} &= \mathbf{1} \\ \boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z} &= \mathbf{1} \\ \boldsymbol{x} + \propto \boldsymbol{y} + \boldsymbol{z} &= \mathbf{2} \end{aligned}$$

has: (i) **no solution** (ii) **unique solution** (iii) **infinitely many solutions**.

b) **Solve** the following **homogeneous linear system**. **Why** <u>this system cannot be solved by Cramer's Rule</u>?

$$\begin{aligned} x - 2y + 3z &= 0 \\ 3x + y - 2z &= 0 \\ 2x - 4y + 6z &= 0. \end{aligned}$$

Question 4: [Marks: 3+3**]**

a) **Show** that {1 - x, 1- x^2, 1+ x+ x^2} is a **basis** of the **vector space** P_2 of all polynomials in real variable x with **degree** $\leq$ **2.**

b) **Let S** = $\{(1, 0, 1, 1), (1, -1, 2, 1), (1, -2, 3, 1)\}$ **generates** the **vector subspace F** of the Euclidean space $\mathbb{R}^4$. **Find** a **basis** of **F** contained in **S** and **show** that $(0, -2, 7, 6) \notin$ **F.**

Question 5: [Marks: 3+3**]**

a) **Let B** = **{(2, 1), (1, 0)}** and **C** = **{(1, -2), (0, 1)}** be **bases** of the Euclidean space $\mathbb{R}^2$ and $v = (1, 2)$. **Find** the **coordinate vector** $[v]_B$ and the **transition matrix** ${}_CP_B$. Then use the transition matrix to **find** $[v]_C$.

b) **Let A** = $\begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 2 & 1 & 1 & 1 & 0 \\ -1 & -2 & 1 & 0 & 1 \\ -2 & -2 & 0 & 1 & 4 \end{bmatrix}$. **Find**:

(i) a **basis** of ***col*** (**A**) (ii) ***rank*** (**A**) (iii) ***nullity*** (**A**).

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