Solutions

جامعة الملك سعود
King Saud University

College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Second Midterm Exam
Academic Year Choose an item.- First Semester

Exam Information معلومات الامتحان			
Course name	Models of Financial Economic		اسم المقرر
Course Code	ACTU-473		رمز المقرر
Exam Date	2024-10-30	1446-04-27	تاريخ الامتحان
Exam Time	01: 00 PM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name	Dr. Dalal Alghanem and Dr. Souhail Chebbi		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 2 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 2 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	Describe some models of option pricing in financial markets			
2	Demonstrate mastery of fundament concepts of option pricing in discrete and continuous cases			
3	Apply option pricing methods to evaluate derivatives			
4	Control risk by using options in a hedging context.			
5	Model the Black Scholes option pricing for derivatives			
6	Demonstrate commitment to actuarial professional and academic values			
7	Prepare students to International Exams			
8				

EXAM COVER PAGE

Exercise 1. [5]

Assume the Black-Scholes framework. You are given that:

(i) The current stock price is 10.
(ii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 2%.
(iii) The stock's volatility is 36%.
(iv) The continuously compounded risk-free interest rate is 5%.

A contingent claim pays $Ln[S(3)]$ at time 3.

Find the time-0 price of the contingent claim.

Exercise 2. [5]

Consider a stock that follows the Black-Scholes model. You are given that:

(i) The current stock price is 100.
(ii) The stock pays no dividends.
(iii) The 95% lognormal prediction for price of the stock four years from now is $(80, 200)$. ($z_{\frac{\beta}{2}} = 1.95996$)

Find the expected rate of appreciation for the stock.

Exercise 3. [5]

Consider a European option with a maturity of 2 years. The payoff is the stock price $S(2)$ if it is greater than 46, and is nothing otherwise. Suppose that the stock price's appreciation rate is 8% and the stock's volatility is 25%. The current stock price is 50.

(a) What is the probability that the payoff after 2 years is strictly positive, as viewed at $t = 0$?
(b) Find the expected payoff of the option, as viewed at $t = 0$.

Exercise 4. [5]

Suppose that S is a non-dividend-paying stock with a current price of 30. Assume the Black-Scholes framework, and that the volatility of the stock is 22%. The continuously compounded risk-free interest rate is 5%. Consider a contingent claim that has a time-0.25 payoff of:

$$Max[0, 30 - \max(S(0.25), 28)]$$

(a) Show that this contingent claim is a bear spread.
(b) Compute the time-0 price of this contingent claim.

Exercise 5. [5]

For a stock, you are given:

(i) The current stock price is 50
(ii) The stock will pay a dividend of 3 six months from now and another dividend of 1 one year from now.
(iii) The prepaid forward price for the delivery of 1 share of the stock at time 1 has a volatility of 0.3.
(iv) The continuously compounded risk-free interest rate is 0.03.

Calculate the price of a put option with strike 55 maturing in 1 year.

Exercise 6. [2 Bonus]

Let $S(t)$ be the price of a non-dividend paying stock at time t. Suppose that the stock follows the Black-Scholes framework, has a volatility of 20%, and an expected rate of appreciation of 6%.

If $Cov[S(1), S(2)] = 156$, find $S(0)$.

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

	Values of z for selected values of Pr(Z<z)						
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Solutions MT2 - ACTU 473
Sem 461

Ex1

$S_0 = 10 \qquad \delta = 0.02 \qquad \sigma = 0.36 \qquad r = 0.05 \qquad T = 3$

$V = e^{-rT} E^*[\text{Payoff}]$. We first compute

$E[\ln(S(3))]$

We know that:

1 $\quad S(T) = S(0) \exp\left[(\alpha - \delta - \sigma^2/2)T + \sigma\sqrt{T}\, Z\right]$

$\Rightarrow$ 1 $\quad \ln[S(T)] = \ln[S(0)] + (\alpha - \delta - \sigma^2/2)T + \sigma\sqrt{T}\, Z$

$\Rightarrow$ 1 $\quad E[\ln S(T)] = \ln[S(0)] + (\alpha - \delta - \sigma^2/2)T$

We deduce

1 $\quad E^*[\ln S(T)] = \ln[S(0)] + (r - \delta - \sigma^2/2)T$

$\Rightarrow$ 1 $\quad V = e^{-0.05\times 3}\left[\ln(10) + \left(0.05 - 0.02 - \frac{0.36^2}{2}\right)\times 3\right]$

$V = 1.891995$

Ex2 We know that:

2 $\begin{cases} 80 = 100 \exp\left[(\alpha - \sigma^2/2)\times 4 - 1.95996\,\sigma\sqrt{4}\right] & (1) \\ 100 = 100 \exp\left[(\alpha - \frac{\sigma^2}{2})\times 4 + 1.95996\,\sigma\sqrt{4}\right] & (2) \end{cases}$

$\frac{(2)}{(1)} \Rightarrow \quad 2.5 = \exp(2 \times 1.95996 \times 2\sigma)$ 1

$\Rightarrow \quad \sigma = 0.116876$ 1

$(1) \Rightarrow \quad 80 = 100\, e^{4\alpha - 0.4854646}$ 1

$\Rightarrow \quad \alpha = 0.0656$

$S_0 = 50 \qquad \alpha - \delta = 8\% \qquad \sigma = 0.25$

$$\text{Pay-off} = \begin{cases} S(2) & \text{if } \quad S(2) \geq 46 \\ 0 & \text{if } \quad \text{not} \end{cases}$$

a) $\text{Pay-off} > 0 \Rightarrow S(2) \geq 46$

$P(S(2) \geq 46) = N(\hat{d}_2)$ 1

$$\hat{d}_2 = \frac{\ln\left(\frac{50}{46}\right) + \left(0.08 - 0.5 \times 0.25^2\right) \times 2}{0.25\sqrt{2}} = 0.51161$$

$\Rightarrow \quad N(\hat{d}_2) = 0.69553$ 1

b) $\text{Pay-off} = S(2) \, I(S(2) \geq 46)$

$E[\text{Pay-off}] = E[S(2) \, I(S(2) \geq 46)]$

$= E[S(2)] \, N(\hat{d}_1)$ 1

$E[S(2)] = S(0) e^{(\alpha - \delta) \times 2} = 50 e^{0.08 \times 2}$

$\hat{d}_2 = \hat{d}_1 - \sigma\sqrt{T} \Rightarrow \hat{d}_1 = \hat{d}_2 + \sigma\sqrt{T}$ 1

$= 0.51161 + 0.25\sqrt{2}$

$\hat{d}_1 = 0.865164$

$\Rightarrow \quad N(\hat{d}_1) = 0.80653$

$\Rightarrow \quad E[\text{Pay-off}] = 50 e^{0.08 \times 2} \times 0.80653$ 1

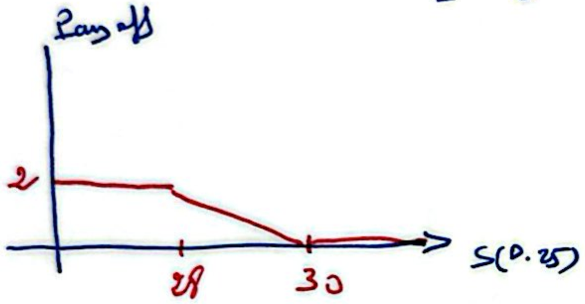
$= 47.3235$

$S_0 = 30 \quad \sigma = 22\% \quad r = 5\%$

$$\text{Payoff} = \text{Max}\left[0, \; 30 - \max\left(S(0.25), 28\right)\right]$$

- If $S(0.25) < 28$ $\quad$ Payoff $= \text{Max}(0, 30-28) = 2$
- If $28 \leq S(0.25) \leq 30$ $\quad$ Payoff $= \text{Max}(0, 30 - S(0.25))$ $= 30 - S(0.25)$
- If $S(0.25) > 30$ $\quad$ Payoff $= \text{Max}(0, 30 - S(0.25))$ $= 0$

Graph



$\Rightarrow$ Bear Spread with Put

Verification: Bear Spread Put = Sell lower Put (28) + Buy higher Put (30)

(2)

Payoff $= -\text{Max}(28 - S_T, 0) + \text{Max}(30 - S_T, 0)$

if $S_T < 28$ $\quad$ Payoff $= -28 + S_T + 30 - S_T = 2$ ✓

if $28 \leq S_T \leq 30$ $\quad$ Payoff $= 30 - S_T$ ✓

if $S_T > 30$ $\quad$ Payoff $= 0$ ✓

We conclude that: $V = -P_1 + P_2$

P_1: ? $\quad k = 28$

$d_1 = 0.79584 \Rightarrow N(-d_1) = 0.21306$

(1) $d_2 = 0.68584 \Rightarrow N(-d_2) = 0.24641$

$\Rightarrow P_1 = 28 e^{-0.05 \times 0.25}(0.24641) - 30(0.21306)$

$P_1 = 0.42197$

P_2? $\quad K = 30$

$d_1 = 0.168636$

$\rightarrow N(-d_1) = 0.43304$

$d_2 = 0.058636$ (1)

$\Rightarrow N(-d_2) = 0.47662$

$P_2 = 1.12978$

$V = -0.42197 + 1.12978$

$V = 0.70781$ (1)

5

$S_0 = 50 \qquad \sigma = 0.3 \qquad r = 0.03 \qquad K = 55 \qquad T = 1$

$$P = F^P_{0,1}(K)\, N(-d_2) - F^P_{0,1}(S)\, N(-d_1)$$

1 $F^P_{0,1}(K) = K e^{-rT} = 55 e^{-0.03} = 53.3745$

2 $F^P_{0,1}(S) = S_0 - PV(Div) = 50 - \left[3e^{-0.03 \times 0.5} + e^{-0.03} \right]$

$= 46.0742$

$$d_1 = \frac{\ln\left(\frac{46.0742}{53.3745}\right) + 0.5 \times 0.3^2}{0.3} = -0.340265$$

$\Rightarrow N(-d_1) = 0.63317$ 1

$d_2 = -0.640265 \Rightarrow N(-d_2) = 0.739$

$\Rightarrow P = 10.27$ 1

Ex6

$$Cov[S(1), S(2)] = Var[S(1)]\, e^{(\alpha-\delta)(2-1)}$$

$$= \left[S(0) e^{0.06}\right]^2 \left(e^{\sigma^2} - 1\right) e^{0.06}$$ 1

$$\Rightarrow 156 = S(0)^2 e^{0.12} \left[e^{0.04} - 1 \right] e^{0.06}$$ 1

$$\Rightarrow S(0) = 56.51$$ 1