

First Midterm Exam
Academic Year 2025-2026 - First Semester

| Exam Information | |
|------------------|----------------------------------------|
| Course name | Introduction to Differential Equations |
| Course Code | Math 280 |
| Date Exam | 2-10-2025 |
| Exam Time | 01: 00 PM |
| Duration Exam | 1H30mns |
| Classroom No. | 71 |
| Instructor Name | Mongi Blel |

| Student Information | |
|----------------------|--|
| Student's Name | |
| ID Number | |
| Section Number | |
| Serial Number | |
| General Instructions | |

Instructor Name

- Your Exam consists of 4 PAGES
- Keep your mobile and smart watch out of the classroom.

This section is ONLY for instructor

| # | Course Learning Outcomes (CLOs) | Related Question(s) | Points | Final Score |
|---|---------------------------------|---------------------|--------|-------------|
| 1 | 1-1 (20 points) | Q(1,2,3) | | |

Question 1 :

Determine $\sup A$, $\sup B$, $\inf A$ and $\inf B$, for

$$A = \left\{ \frac{(-1)^n}{2n} - \frac{(-1)^m}{3m}; \quad n, m \in \mathbb{N}; \right\},$$

$$B = \{x \in \mathbb{R} : \text{the sequence } (x^n)_n \text{ is convergent}\}.$$

Question 2 :

Consider the sequence $(x_n)_n$ defined by $x_1 = 0$, and $x_{n+1} = \sqrt{2 + x_n}$ for all $n \in \mathbb{N}$.

1. Prove that the sequence $(x_n)_n$ is increasing and $x_n \leq 2$ for all $n \in \mathbb{N}$.
2. Deduce that the sequence $(x_n)_n$ is convergent and determine its limit.

Question 3 :

1. Give a sequence $(x_n)_n$ convergent to 0 but the series $\sum_{n \geq 1} x_n$ is divergent.

2. Study the convergence of the following series

(a) $\sum_{n \geq 1} \frac{(-1)^n \ln n}{n}$. (b) $\sum_{n \geq 1} \frac{n2^n}{n!}$. (c) $\sum_{n \geq 1} \frac{n^n}{2^{2n}}$.

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