Solution of the first examination Math 280 Semester I , 2025

Question 1 :

$B = (-1, 1]$. $\sup A = \frac{7}{12}$, $\inf A = -\frac{2}{3}$, $\sup B = 1$, and $\inf B = -1$.

Question 2 :

1. $x_n \geq 0$, for all $n \in \mathbb{N}$. $x_1 = 1$, then $x_1 \geq x_0$. Assume that $x_{n-1} \leq x_n$, hence $x_{n+1} = \sqrt{2 + x_n} \geq \sqrt{2 + x_{n-1}} = x_n$. Then the sequence $(x_n)_n$ is increasing.
   Also we have $x_0 \leq 2$. Assume that $x_n \leq 2$. We have $x_{n+1} = \sqrt{2 + x_n} \leq 2$. Then $x_n \leq 2$ for all $n \in \mathbb{N}$.

2. The sequence $(x_n)_n$ is increasing and bounded above then it is convergent. Its limit is 2.

Question 3 :

1. $x_n = \frac{1}{n}$.

2. the series $\displaystyle\sum_{n\geq 1} \frac{(-1)^n \ln n}{n}$ is alternate and the sequence $(\frac{\ln n}{n})_n$ is decreasing, then the series is convergent

3. $\displaystyle\lim_{n\to+\infty} \frac{(n+1)2^{n+1}n!}{(n+1)!n2^n} = \lim_{n\to+\infty} \frac{2}{n} = 0$, then the series $\displaystyle\sum_{n\geq 1} \frac{n2^n}{n!}$ is convergent.

4. $\displaystyle\lim_{n\to+\infty} \frac{2^{2n}(n+1)^{n+1}}{n^n 2^{2n+2}} = \lim_{n\to+\infty} \frac{n+1}{4}\left(1 + \frac{1}{n}\right)^n = +\infty$, then the series $\displaystyle\sum_{n\geq 1} \frac{n^n}{2^{2n}}$ is divergent.