461

Solutions

جـامـعـة
الملك سعود
King Saud University

College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

~~Second~~ Final Midterm Exam
Academic Year Choose an item.- SecondSemester

| Exam Information معلومات الامتحان | | | |
|---|---|---|---|
| Course name | Models of Financial Economic | | اسم المقرر |
| Course Code | ACTU-473 | | رمز المقرر |
| Exam Date | 2024-10-03 | 1446-03-30 | تاريخ الامتحان |
| Exam Time | 01: 00 PM | | وقت الامتحان |
| Exam Duration | 2 hours | ساعتان | مدة الامتحان |
| Classroom No. | | | رقم قاعة الاختبار |
| Instructor Name | Dr. Dalal Alghanem and Dr. Souhail Chebbi | | اسم استاذ المقرر |

| Student Information معلومات الطالب | | |
|---|---|---|
| Student's Name | | اسم الطالب |
| ID number | | الرقم الجامعي |
| Section No. | | رقم الشعبة |
| Serial Number | | الرقم التسلسلي |

**General Instructions:**

- Your Exam consists of 2 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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**تعليمات عامة:**

- عدد صفحات الامتحان 2 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

*This section is ONLY for instructor*

| # | Course Learning Outcomes (CLOs) | Related Question (s) | Points | Final Score |
|---|---|---|---|---|
| 1 | Describe some models of option pricing in financial markets | | | |
| 2 | Demonstrate mastery of fundament concepts of option pricing in discrete and continuous cases | | | |
| 3 | Apply option pricing methods to evaluate derivatives | | | |
| 4 | Control risk by using options in a hedging context. | | | |
| 5 | Model the Black Scholes option pricing for derivatives | | | |
| 6 | Demonstrate commitment to actuarial professional and academic values | | | |
| 7 | Prepare students to International Exams | | | |
| 8 | | | | |

EXAM COVER PAGE

**Exercise 1. [5]**

For a non-dividend paying stock, you are given:

(i) The current price of the stock is 50.
(ii) In one year, the stock will either go up to 55 or down to 46.
(iii) The continuously compounded risk-free interest rate is 4%.

Suppose that the current price of a 1-year 51-strike European call on the stock is 2.60. Find an arbitrage strategy and the arbitrage profit.

**Exercise 2. [5]**

For a stock, you are given:

(i) The current stock price is 90.
(ii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 2%.
(iii) The stock's volatility is 20%.
(iv) You use a three-period binomial forward tree to model the stock price movement. The length of each period is 3 months.

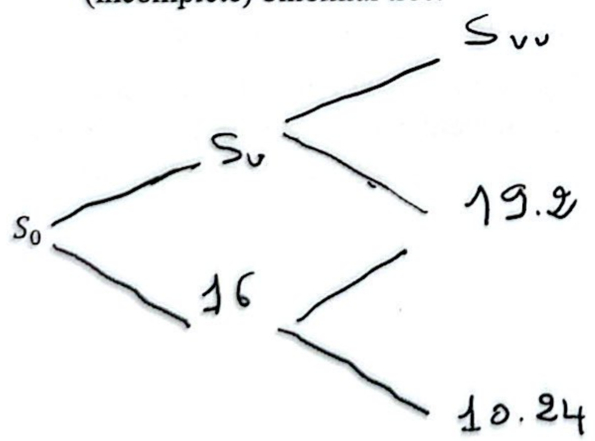
You are also given that the continuously compounded risk-free interest rate is 10%. Consider a 9-month 90-strike American put on the stock.

Calculate the risk-neutral probability that the option will be exercised before maturity.

**Exercise 3. [5]**

For a non-dividend paying stock, you are given:

(i) The stock price movement can be modeled by the following (incomplete) binomial tree.



(ii) The length of each period is 1 year.
(iii) The continuously compounded risk-free interest rate is 5%.

Calculate the price of a 2-year 30-strike American call on the stock.

**Exercise 4. [5]**

You use a one-period forward tree to price a 1-year contingent claim on a non-dividend paying stock.

(i) The current stock price is 75.
(ii) The continuously compounded annual expected return on the stock is 10%.
(iii) The stock's volatility is 24%.
(iv) The continuously compounded risk-free interest rate is 6%.
(v) The option pays 3 if the stock price goes up after 1 year and pays 0 otherwise.

Calculate the appropriate discount rate for the contingent claim in the real world.

**Exercise 5. [5]**

You are given:

(i) The spot price of the Euro is 0.95 US dollars.
(ii) The Euro-Dollar exchange rate has a volatility of 15%.
(iii) The euro-denominated and dollar-denominated continuously compounded risk-free interest rates are 4% and 6% respectively.

Calculate the price (in Euro) of a call option to buy 95 Dollars with 100 Euro six months from now by constructing a one-period forward tree to model the movement of the exchange rate.

# MT1 - ACTU 473 - 461
## Solutions

**Ex 1**

We form the replicating portfolio of the option:

$$S_0 = 50 \begin{cases} 55 \\ 46 \end{cases} \qquad C_0 \begin{cases} 4 = C_u \\ 0 = C_d \end{cases}$$

$$\Delta = \frac{4-0}{55-46} = 4/9 \qquad B = \frac{e^{-0.04}\left(-\frac{46}{50}\times 4\right)}{\frac{9}{50}}$$

2

$$B = -19.6428$$

So, the no-arbitrage price is $C_0 = S_0\Delta + B$

$$C_0 = 2.579416$$

The given price $C_0 = 2.60 > 2.579416 = S_0\Delta + B$

Arbitrage Strategy:

At $t=0$

| | | |
|---|---|---|
| - | Sell Call option | 2.60 |
| - | Take a loan | 19.64 |
| - | Buy $\Delta$ (shares) of stocks | -22.2 |
| | Pay off at $t=0$: | 0.04 |

3

At $t=1$

- Holder of $\Delta$ (shares) of stocks $\begin{cases} 24.44 \\ 20.44 \end{cases}$
- Execute the call option $\begin{cases} -4 \\ 0 \end{cases}$
- Clear the loan $-20.44$

Payoff at $t=1$: $\begin{cases} 0 \\ 0 \end{cases}$

Arbitrage profit $= 0.04$

2

$S_0 = 90 \qquad \delta = 2\% \qquad \sigma = 20\%$

$u = e^{(r-\delta)h + \sigma\sqrt{h}} = 1.1275$

$d = e^{(r-\delta)h - \sigma\sqrt{h}} = 0.9231$

$p^* = \dfrac{1}{1 + e^{\sigma\sqrt{h}}} = 0.47502$

$$\begin{tikzcd}[column sep=small, row sep=tiny]
 & & & 128.99965 \\
 & & 114.41242 \arrow[ur] \arrow[dr] & \\
 & 101.47472 \arrow[ur] \arrow[dr] & & 105.61598 \\
90 \arrow[ur] \arrow[dr] & & 93.67297 \arrow[ur] \arrow[dr] & \\
 & 83.08047 \arrow[ur] \arrow[dr] & & 86.47105 \\
 & & 76.69294 \arrow[ur] \arrow[dr] & \\
 & & & 70.79651
\end{tikzcd}$$

$\Rightarrow \quad C_{ddd} = 19.20349 \qquad C_{udd} = 3.52895 \qquad C_{uud} = C_{uuu} = 0$

– At the node dd : $C_{dd} = \text{Max}\left[13.30706,\ e^{-0.025}\left(p^* C_{udd} + (1-p^*) C_{ddd}\right)\right]$

$= 13.30706 \rightarrow$ the option is exercised

– $C_{ud} = e^{-0.025}\left(0\, p^* + 3.52895\,(1-p^*)\right) = 1.80688$

$\rightarrow$ not exercised

– $C_{uu} = 0 \ , \quad C_u = 0$

– $C_d = \text{Max}\left[6.91953,\ e^{-0.025}\left(1.80688\, p^* + 13.30706\,(1-p^*)\right)\right]$

$= 7.65056 \rightarrow$ not exercised

$\Rightarrow$ There is only 1 node for which the option is exercised : dd

$\Rightarrow$ The required probability is $(1-p^*)^2 = 0.2756$

1

Ex 3

Recall that for non-dividend paying stock, an American call has the same price as an otherwise identical European call.

2

$u = \frac{19.2}{16} = 1.2$ $\qquad$ $d = \frac{10.24}{16} = 0.64$

$p^* = \frac{e^{0.05} - 0.64}{1.2 - 0.64} = 0.7344$

$S_0 = \frac{S_d}{d} = \frac{16}{0.64} = 25$ $\qquad$ $S_u = 25u = 30$

$S_{uu} = 30u = 36$

1/2

$\Rightarrow \quad C_{uu} = 6 \qquad C_{ud} = 0 \qquad C_{dd} = 0$

$\Rightarrow \quad C_0 = e^{-2rh} E^*[C_{2h}] = e^{-2\times 0.05}\left[(p^*)^2 \times 6\right]$

$C_0 = 2.9282$ $\qquad$ 2

Ex4

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{0.3}$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{-0.18}$$

$$\Rightarrow \quad 2 \quad \begin{cases} \Delta = e^{-\delta h} \dfrac{C_u - C_d}{S_u - S_d} = \dfrac{3 - 0}{75\left(e^{0.3} - e^{-0.18}\right)} = 0.07773 \\[2ex] B = e^{-rh} \dfrac{uC_d - dC_u}{u - d} = e^{-0.06} \dfrac{\left(0 - e^{-0.18} \times 3\right)}{e^{0.3} - e^{-0.18}} = -4.585962 \end{cases}$$

2 $$e^{\gamma} = \frac{75\,\Delta}{75\Delta + B} e^{\alpha} + \frac{B}{75\Delta + B} e^{r}$$

$$e^{\gamma} = 1.2649 \quad \Rightarrow \quad \gamma = 0.2350 \quad 1$$

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