

First Semester

Second Quiz

King Saud University

(without calculators)

Time: 20 mins.

College of Science

Sunday 11-5-1447

240 Math

Math. Department

Name:

ID no.:

Q1: Show that the set $S=\{(1,1,1),(1,2,1),(1,3,3)\}$ forms a basis for \mathbb{R}^3 . (3 marks)

Q2: Let $V=\mathbb{R}^2$ which has two operations are defined, addition:

For all $(u_1,u_2),(v_1,v_2)\in \mathbb{R}^2$: $(u_1,u_2)+(v_1,v_2)=(u_1+v_1,u_2+v_2)$

and scalar multiplication: For all $(u,v)\in \mathbb{R}^2$ and $k\in \mathbb{R}$: $k(u,v)=(k^3u,kv)$

Show that V is **not** a vector space. (2 marks)

Solution

Q1 Since

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 3 \end{vmatrix} \begin{matrix} (-1)R_{12} \\ = \\ (-1)R_{13} \end{matrix} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{vmatrix} = 2 \neq 0$$

So, S is a basis for \mathbb{R}^3 .

Q2: The axiom $(k+m)u=ku+mu$ is not true here. Take $k=m=1$ and $u=(1,1)$. Then

$$(1+1)(1,1)=2(1,1)=(8(1),2(1))=(8,2)$$

but

$$1(1,1)+1(1,1)=(1,1)+(1,1)=(2,2) \neq (8,2)$$

So, it is not a vector space.