

Second Semester

Second Quiz

King Saud University

(without calculators)

Time: 20 mins.

College of Science

Sunday 8-10-1446

240 Math

Math. Department

Name:

ID no.:

Q1: Show that the set $S=\{(1,1,1),(1,2,1),(1,3,2)\}$ forms a basis for $\mathbb{R}^3$. (3 marks)

Q2: Let $V=\mathbb{R}^2$ which has two operations are defined, addition:

For all $(u_1, u_2), (v_1, v_2) \in \mathbb{R}^2$: $(u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$

and scalar multiplication: For all $(u, v) \in \mathbb{R}^2$ and $k \in \mathbb{R}$: $k(u, v) = (k^3 u, kv)$

Show that V is not a vector space. (2 marks)

Solution

Q1 Since

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix} \xrightarrow[(-1)R_{13}]{(-1)R_{12}} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0$$

So, S is a basis for $\mathbb{R}^3$.

Q2: The axiom $(k+m)u=ku+mu$ is not true here. Take $k=m=1$ and $u=(1,1)$. Then
 $(1+1)(1,1)=2(1,1)=(8(1),2(1))=(8,2)$

but

$$1(1,1)+1(1,1)=(1,1)+(1,1)=(2,2) \neq (8,2)$$

So, it is not a vector space.