

First Semester

Second Quiz

King Saud University

(without calculators)

Time: 20 mins.

College of Science

Thursday 21-4-1446

240 Math

Math. Department

Name:

ID no.:

Q1: Let V be any nonempty set which has two operations are defined: addition and scalar multiplication. State 6 axioms that should be satisfied by all scalars and all objects in V that make V a vector space. (3 marks)

Q2: Let $V = \mathbb{R}^2$ which has two operations are defined, addition:

For all $(u_1, u_2), (v_1, v_2) \in \mathbb{R}^2$: $(u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$

and scalar multiplication: For all $(u, v) \in \mathbb{R}^2$ and $k \in \mathbb{R}$: $k(u, v) = (k^2 u, k^2 v)$

Show that V is not a vector space. (2 marks)

Solution

Q1 For all $u, v, w \in V$ and $k, m \in \mathbb{R}$:

- 1- $u+v \in V$
- 2- $u+v=v+u$
- 3- $u+(v+w)=(u+v)+w$
- 4- there is a zero vector 0 in V such that $u+0=u$ for all $u \in V$
- 5- for each vector u in V , there is a negative vector $-u$ such $u+(-u)=0$
- 6- $ku \in V$
- 7- $k(u+v)=ku+kv$
- 8- $(k+m)u=ku+mu$
- 9- $K(mu)=(km)u$
- 10- $1u=u$

Q2: The axiom $(k+m)u=ku+mu$ is not true here. Take $k=m=1$ and $u=(1,1)$. Then

$$(1+1)(1,1)=2(1,1)=(4(1),4(1))=(4,4)$$

but

$$1(1,1)+1(1,1)=(1,1)+(1,1)=(2,2) \neq (4,4)$$

So, it is not a vector space.