

Second Semester

First Quiz

King Saud University

(without calculators)

Time: 20 mins.

College of Science

Thursday 30-7-1446

240 Math

Math. Department

Name:

ID no.:

Q1: If $A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$, $B = [1 \ 1 \ 3]$ and $p(x) = x^2 - 1$, then find the following:

(a) $\text{tr}(A^{-1})$ (2 marks).

(b) $B^T B$ (1 mark).

(c) $p(A)$ (2 marks).

Solution

Q1:

(a)

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix} \Rightarrow \text{tr}(A^{-1}) = \frac{1}{2} \text{tr} \left(\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix} \right) = \frac{1}{2} (1+4) = \frac{5}{2}$$

(b)

$$B^T B = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{bmatrix}$$

(c)

$$\begin{aligned} p(A) &= A^2 - I = AA - I = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 18 & 10 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 5 & 2 \end{bmatrix} \end{aligned}$$