EXERCISE SHEET #2

MATH 111 and MATH 106 Exercises

Note: These exercises will be discussed in the tutorial lecture, but you are advised to solve the exercises at the end of each section in the book.

Chapter 5

5.1:

1. Let R be the region bounded by the graphs of the following functions. Sketch R and find its area.

(a) $y = x^2 - 1, y = \frac{1}{5}x^2$

(b) $y = 9 - x^2, y = 3 - x$

(c) $y = x^2, y = x - 1,\ 2 \le x \le 4$

(d) $y = x^2, y = 2 - x^2$

(e) $y = x^2, y = \sqrt{x}$

(f) $y = \sin x, y = \cos x,\ 0 \le x \le \pi$

(g) $y = e^x, y = e^{-x}, y = 0,\ -1 \le x \le 1$

(h) $2y^2 = x + 4, y^2 = x$

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2. Sketch and find the area of the region bounded by the given curves. Choose the variable of integration so that the area is written as a single integral

(a) $y = 2x, y = 3 - x^2, x = 0$ in the first quadrant

(b) $x = 3y, x = 2 + y^2$

5.2:

1. **Let R be the region bounded by the graphs of the following functions. Sketch R and find the volume of the solid resulting by revolving R about the indicated axis**

(a) $y = x^2, y = 4 - x^2,\quad x$ -axis

(b) $y = x, x + y = 4, x = 0,\quad x$ -axis

(c) $y = x, x + y = 4, x = 0,\quad y$ -axis

(d) $y = x^2, y = 2,\quad y$ -axis

(e) $x^2 = y - 2, 2y - x = 2, x = 0, x = 1,\quad x$ -axis

(f) $y = x^2, y = x + 2,\quad x$ -axis

(g) $y = e^x, x = 0, x = 2, y = 0,\quad y$ -axis

5.3:

1. Let R be the region bounded by the graph of $y = \sqrt{x - 1},\ x = 10$ and $y = 0.$ Sketch R and find the volume of the solid generated by revolving R about the y -axis using

(a) Method of washers.

(b) Method of cylindrical shells.

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2. Let R be the region bounded by $y = \sqrt{x}$, $x -$ axis and the line $x = 4$. *Sketch R* and *set up the integral for the volume* of the solid resulting by revolving R about

(a) The $x -$ axis.

(b) The $y -$ axis

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3. Sketch the region, draw in a typical shell, identify the radius and height of each shell and compute the volume

(a) The region bounded by $y = x, y = -x,$ and $x = 1,$ revolved about the $y -$ axis.

(b) The region bounded by $y = x^2$ and $y = 2 - x^2$ revolved about the $y -$ axis.

(c) The region bounded by $x = y^2$ and $x = 4$ revolved about the $x -$ axis.

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4. Use the best method available to find each volume

(a) The region bounded by $y = x + 2, y = -x - 2,$ and $x = 0$ revolved about

(i) The $x -$ axis (ii) The $y -$ axis

(b) The region bounded by $y = 2 - x^2, y = x$ and $x = 0$ in the first quadrant, revolved about

(i) The $x -$axis **(ii)** The $y -$axis

5.4:

1. Compute the arc length of $f(x) = 2x + 1$ on $[0,2]$
2. Compute the arc length of $f(x) = \frac{2}{3}x^{\frac{3}{2}}$ on $[0,1]$
3. Compute the arc length of $f(x) = \frac{1}{4}(e^{2x} + e^{-2x})$ on $[0,1]$
4. Compute the arc length of $f(x) = \frac{1}{6}x^3 + \frac{1}{2x}$ on $[1,3]$
5. Compute the arc length of $y = \ln(\sin x)$ from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{2}$.
6. Compute the surface area of the solid resulting when $f(x) = 2\sqrt{x+1}$ is revolved about the x-axis on [0,3]
7. Set up the integral for finding the surface area of the solid obtained by revolving the curve $y = \sqrt{4 - x^2}, x \in [-2,2]$ about the x-axis.
8. Set up the integral for finding the surface area of the solid obtained by revolving the curve $y = \ln x, x \in [1,2]$ about the x-axis.

   Set up the integral for finding the surface area of the solid obtained by revolving the curve $y = 2x - x^2, x \in [0,2]$ about the x-axis.

Chapter 9

9.1:

1. **Sketch the plane curve defined by the given parametric equations and find a corresponding x - y equation for the curve**

   (a) $x = 3\sin t, y = 2\cos t,\ 0 \le t \le 2\pi$

   (b) $x = 2 + \cos(2t),\ \ y = -1 + \sin(2t),\ \ 0 \le t \le \pi$

(c) $x = t^2, y = 2\ln t,\ t > 0$

(d) $x = 3 + 2t,\ y = 1 - t,\ \ 0 \le t \le 1$

(e) $x = 1 + t, y = t^2 + 2,\ t \in \Re$

(f) $x = e^t, y = e^{2t},\ t \in \Re$

2. **Find the parametric equations describing the following**

(a) The line segment from $(3,1)$ to $(1,3)$

(b) The line segment from $(4,-2)$ to $(2,-1)$

(c) The portion of the parabola $y = x^2 + 1$ from $(1,2)$ to $(2,5)$

(d) The circle of radius 2 centered at $(-1,3)$ drawn counterclockwise

3. **Find all points of intersection of the two curves**

(a) $x = t^2, y = t + 1$ and $x = 2 + s, y = 1 - s$

(b) $x = t + 3, y = t^2$ and $x = 1 + s, y = 2 - s$

9.4:

1. **Sketch the graph of the following polar equations**

(a) $r = 3$

(b) $r = -3$

(c) $\theta = \dfrac{\pi}{6}$

(d) $\theta = -\dfrac{\pi}{6}$

(e) $r = 3\cos\theta$

(f) $r = 6\sin\theta$

2. **Plot the given polar points (r,θ) and find their rectangular representation (x, y)**

(a) $(5,0)$

(b) $(5,\pi)$

(c) $(3,-\pi)$

(d) $(-3,\pi)$

(e) $\left(3, \dfrac{\pi}{2}\right)$

(f) $\left(-3, \dfrac{3\pi}{2}\right)$

(g) $\left(2, \frac{-\pi}{3}\right)$

(h) $(-2,1)$

3. Find all polar coordinate representations of the given rectangular point

(a) $(1,2)$

(b) $(-1,1)$

(c) $(3,-3)$

(d) $(-1,-\sqrt{3})$

4. Sketch the graph of the following polar equations and find the area of the region bounded by such a graph, where $\theta \in [0,2\pi]$

(a) $r = \cos(2\theta)$

(b) $r = \sin(2\theta)$

(c) $r = \cos(3\theta)$

(d) $r = -2 + 2\cos\theta$

(e) $r = 4 + 2\cos\theta$

(f) $r = 3 + 6\cos\theta$

(g) $r = 2 - 4\sin\theta$

(h) $r = 3 + 2\sin\theta$

5. Find a polar equation corresponding to the following rectangular equations

(a) $\frac{y}{x} = 1, x \neq 0$.

(b) $4x\sqrt{x^2 + y^2} = 6y, (x, y) \neq (0,0)$

6. Find a rectangular equation corresponding to the following polar equations

(a) $r^2 \sin(2\theta) = 1$

(b) $r = 2\sec\theta$

7. Solve the following equations for θ, $\theta \in [0,2\pi]$

(a) $\cos\theta = \sin\theta$

(b) $1 - 2\sin\theta = 2\cos\theta$