King Saud University: Mathematics Department MATH-254
First Semester 1446-47 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:———————————— **I.D. No.** ————————-

Name of the Teacher:———————————— **Section No.** ————————

Note: Check the total number of pages are Seven (7).

(**11 Multiple choice questions and Three (3) Full questions**)

The Answer Tables for Q.1 to Q.11 : Marks: 2 for each one ($2 \times 11 = 22$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10	11
a,b,c,d											

Quest. No.	Marks Obtained	Marks for Question
Q. 1 to Q. 11		22
Q. 12		6
Q. 13		6
Q. 14		6
Total		40

Question 1: The value of k which insures rapid convergence of $x_{n+1} = x_n + k(x_n^2 - 5)$ to $\alpha = \sqrt{5}$ is:

(a) $\dfrac{1}{2\sqrt{5}}$ **(b)** $-\dfrac{1}{2\sqrt{3}}$ **(c)** $-\dfrac{1}{2\sqrt{5}}$ **(d)** None of These

Question 2: The first approximation using Secant method of the intersection of $f_1(x) = x^3 + 2x - 1$ and $f_2(x) = \sin x$ with $x_0 = 0.5$ and $x_1 = 0.55$ is:

(a) 0.6608 **(b)** 0.6806 **(c)** 0.8606 **(d)** None of These

Question 3: Let $A = \begin{pmatrix} -4 & 6 \\ -2 & 2 \end{pmatrix}$, then the matrix L of the LU factorization using Crout's method is:

(a) $L = \begin{pmatrix} -4 & 0 \\ -2 & -1 \end{pmatrix}$ **(b)** $L = \begin{pmatrix} 1 & 0 \\ -1/2 & 1 \end{pmatrix}$ **(c)** $L = \begin{pmatrix} 4 & 0 \\ 2 & -1 \end{pmatrix}$ **(d)** None of These

Question 4: The first approximation for solving linear system $A\mathbf{x} = [1, 3]^T$ using Jacobi iterative method with $A = \begin{pmatrix} -4 & 5 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{x}^{(0)} = [0.5, 0.5]^T$ is:

(a) $[0.375, 1.250]^T$ **(b)** $[1.375, 1.315]^T$ **(c)** $[1.375, 1.250]^T$ **(d)** None of These

Question 5: If $x^* = [0.5, 0.0]^T$ is an approximate solution for the system $2x - y = 1$, $x + y = 2$, then the l_∞-norm of the corresponding residual vector is:

(a) 0.25 **(b)** 1.5 **(c)** 0.5 **(d)** None of These

Question 6: Using data points: $(0, -2), (0.1, -1), (0.15, 1), (0.2, 2), (0.3, 3)$, if $\max\limits_{0 \le x \le 0.3} f^{(5)}(x) = 1$, then the error bound in approximating $f(0.25)$ by using a fourth degree interpolating polynomial is bounded by:

(a) 1.56×10^{-6} **(b)** 7.8×10^{-7} **(c)** 7.8×10^{-8} **(d)** None of These

Question 7: Using linear spline which interpolates $f(2.5)$ using data: $(1, 35), (2, 40), (3, 65), (4, 72)$ is:

(a) 62.50 **(b)** 50.50 **(c)** 52.50 **(d)** None of These

Question 8: If $f(x) = x^2 + \cos x$, then best approximation of $f'(1)$ with stepsize $h = 0.1$ using three-point central difference formula is:

(a) 1.1585 **(b)** 1.1599 **(c)** 1.1605 **(d)** None of These

Question 9: If $f(0) = 3, f(1) = \dfrac{\alpha}{2}, f(2) = \alpha$, and Simpson's rule for $\displaystyle\int_0^2 f(x)\ dx$ gives 2, then the value of α is:

(a) 0.5 **(b)** 2.0 **(c)** 1.0 **(d)** None of These

Question 10: Given $xy' + y = 1,\ y(1) = 0$, the approximate value of $y(2)$ using Euler's method when $n = 2$ is:

(a) 0.6667 **(b)** 0.3333 **(c)** 0.1667 **(d)** None of These

Question 11: The absolute error by using the Taylor's method of order 2 of $y(1)$ where $4y' - y = 0,\ y(0) = 1,\ n = 2$, and exact solution $y(x) = e^{x/4}$, is:

(a) 0.1512 **(b)** 0.0080 **(c)** 0.0008 **(d)** None of These

Question 12: If $f(x) = \ln(x+2)$ and $x_0 = -1.5, x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3, x_5 = 4.5$, then find the best approximation of $\ln(3.5)$ by the cubic Newton's polynomial using approximation by quadratic Newton's polynomial equal to 1.2573. Compute the absolute error and an error bound for the approximation of $\ln(3.5)$ by the cubic Newton's polynomial.

<u>Question 13:</u> Let $f(x) = \dfrac{3^x}{x}$. Find the approximation of $f''(x)$ at $x = 3$, taking $h = 0.1$ using three-point central difference formula. Compute the absolute error and an error bound for your approximation if $M = \max_{2.9 \leq x \leq 3.1} |f^{(4)}| = 6.1022$. How many subintervals required to obtain the approximate value of $f''(3)$ within the accuracy 10^{-4}.

Question 14: Find the approximation of $\int_1^2 f(x)\, dx$, by the best composite integration rule using the following table:

x	1.0	1.11	1.2	1.32	1.4	1.5	1.6	1.73	1.8	1.9	2.0
$f(x)$	0.3679	0.3658	0.3614	0.3526	0.3452	0.3347	0.3230	0.3067	0.2975	0.2842	0.2707

The function tabulated is $f(x) = xe^{-x}$, compute an error bound and the number of subintervals to approximate the given integral to an accuracy of at least 10^{-6} ?