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Question Number	I	II	III	IV	Total
Mark					

[1] [4 Points]

1. If $w = \cos(x - y) + \ln(x + y)$, **show** that $w_{xx} = w_{yy}$

2. If $w = x^2 + e^{y^2+z}$, **Find** $\frac{\partial^3 w}{\partial z \partial y^2}$

3. If $z = f(x, y)$ is determined **implicitly** by the equation $xz^2 + x^2y - 4y^2z + 3y = 2$. **Find** $\frac{\partial z}{\partial x}$

[II] [6 Points]

1. Let $w = f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$

(a) **Find** $f_x(0,0)$ and $f_y(0,0)$

(b) **Find** Δw at $(0,0)$

(c) **Discuss** the differentiability of f at $(0,0)$

2. Use the **differentials** to approximate the change in $w = f(x, y) = x - 3x^4y^2 + 2y^3$
From $(2,3)$ to $(2.02, 2.99)$

[III] [6 Points]

1. Find the **extrema** and **saddle points** of $f(x, y) = x^4 - 4xy + 2y^2$

2. Use **Lagrange multipliers** to find the **extrema** of $f(x, y, z) = x^2 + y + z$

subject to the constraint $2x^2 + 2y^2 + z^2 = 2$

[IV] [4 Points]

Let $w = f(u, v)$ where $u = x - 5t$ and $v = x + 5t$. Find $\frac{\partial^2 w}{\partial t^2}$ and $\frac{\partial^2 w}{\partial x^2}$

Good Luck 😊