King Saud University

Department of Mathematics

1st Semester 1445 H

MATH 201 (Differential and Integral Calculus)

Final Exam

Duration: 3 Hours

Name		ID	

Question Number	I	II	III	IV	V	VI	Total
Mark							

[I] Let $f(x, y) = e^{2x} \sin(xy)$. If $g(u, v) = f(u + v, u - v)$, **Find** g_{uv} [4 *Points*]

[II] Let $f(x,y) = \begin{cases} \dfrac{y(x+1)^2 + y^2 \sin(\pi x)}{(x+1)^2 + y^2}, & (x,y) \neq (-1,0) \\ 0, & (x,y) = (-1,0) \end{cases}$ [7 *Points*]

1. **Show** that f is continuous at the point $(-1,0)$

2. **Find** $f_y(-1,0)$

3. **Find** $f_x(0,1)$

[III] Use **Lagrange multipliers** to find the **extrema** of $f(x, y, z) = x^2 + (y - 2)^2 + (z - 3)^2$
subject to the constraint $x^2 + 2y + 2z = 22$ [5 *Points*]

1. **Reverse** the order and **Evaluate** the integral $\int_0^4 \int_{\sqrt{y}}^2 3\sqrt{1 + x^3}\, dx\, dy$

2. **Find** the **volume** of the solid outside the cone $z = 6\sqrt{x^2 + y^2}$ and inside the cylinder $x^2 + y^2 = 4$ that is bounded below by the plane $z = 0$

3. **Evaluate** the integral $\iiint_Q 2z\, dv$ where Q is the solid bounded above by the sphere $x^2 + y^2 + z^2 = 9$ and below by the plane $z = 0$

[V] Determine whether the following series is **absolutely convergent**, **conditionally convergent** or **divergent**. **Justify** your answer. [7 *Points*]

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2+n}}$

2. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{1+n\sqrt{n}}$

3. $\sum_{n=1}^{\infty}(-1)^n \frac{n^2+1}{(2n-3)^2}$

1. Find the **interval** and the **radius** of **convergence** of the power series

$$\sum_{n=1}^{\infty} \frac{(2x-6)^n}{n5^n}$$

2. **Find** a power series representation for $f(x) = e^{1+2x}$

3. Find the **Taylor series** for $f(x) = x\sin 2x$ and use it to approximate $\int_0^{0.5} x \sin 2x \ dx$ to 3- decimal places

Good Luck ☺