MATH 201 EXERCISES (Calculus, Classic Edition by Swokowski)
16.1 1, 3, 5, 8, 9, 14, 21, 22, 23, 24, 37
$16.2 \quad 3,5,6,9,12,14,16,19,20,25,26,28,29,37,38,42$
Find the following limits, if they exist:
(1) $\lim _{(x, y) \rightarrow(1,2)} \frac{x y-2 x-y+2}{x^{2}+y^{2}-2 x-2 y+5}$
(2) $\lim _{(x, y) \rightarrow(1,1)} \frac{x-y^{4}}{x^{3}-y^{4}}$
(3) $\lim _{(x, y) \rightarrow(0,0)} \frac{3 x y}{5 x^{4}+2 y^{4}}$
(4) $\lim _{(x, y) \rightarrow(0,0)} \frac{3 x^{3}-2 x^{2} y+3 y^{2} x-2 y^{3}}{x^{2}+y^{2}}$
(5) $\lim _{(x, y) \rightarrow(0,0)} \frac{10 x y}{5 x^{3}+2 y^{3}}$
(6) $\lim _{(x, y) \rightarrow(0,1)} \frac{x+(y-1)^{3}}{\sqrt{x^{2}+(y-1)^{2}}}$
(7) $\lim _{(x, y, z) \rightarrow(0,0,0)} \frac{x y+y z+z x}{x^{2}+y^{2}+z^{2}}$
(8) $\lim _{(x, y) \rightarrow(0,0)} \frac{x y^{3}}{x^{3}+y^{6}}$
(9) $\lim _{(x, y, z) \rightarrow(0,0,0)} \frac{y^{3}+x^{3} \sin z^{3}}{x^{2}+y^{2}+z^{2}}$
(10) $\lim _{(x, y) \rightarrow(2,1)} \frac{(y-1)(x-2)^{2}}{(y-1)^{3}+(x-2)^{3}}$
(11) $\lim _{(x, y) \rightarrow(0,0)} \frac{3 x^{2} y}{x^{4}+y^{2}}$
(12) $\lim _{(x, y, z) \rightarrow(0,0,0)} \frac{x y^{2}}{x^{2}+y^{2}+z^{2}}$
(13) $\lim _{(x, y) \rightarrow(1,1)} \frac{3 x^{3}+x y^{2}-3 x y-y^{3}}{x^{2}-y^{2}}$
(14) $\lim _{(x, y) \rightarrow(0,0)} \frac{x}{y^{2}}-\frac{x}{e^{y^{2}}}$
(15) $\lim _{(x, y) \rightarrow(0,0)} \frac{x y}{\sqrt{x^{2}+y^{2}}}$
(16) $\lim _{(x, y, z) \rightarrow(0,0,0)} f(x, y, z)$ where $f(x, y, z)=\left\{\begin{array}{cl}\frac{3 x y z}{x^{2}+y^{2}+z^{2}}, & (x, y, z) \neq(0,0,0) \\ 0 & (x, y, z)=(0,0,0)\end{array}\right.$

Discuss the continuity of the following functions on their domain
(1) $f(x, y)=\left\{\begin{array}{cc}\frac{x^{3}+y^{3}}{x^{2}+y^{2}}, & (x, y) \neq(0,0) \\ 0 & (x, y)=(0,0)\end{array}\right.$
(2) $f(x, y)=\left\{\begin{array}{cc}\frac{x^{2} y}{x^{4}+y^{2}}, & (x, y) \neq(0,0) \\ 0 & (x, y)=(0,0)\end{array}\right.$
(3) $f(x, y)=\left\{\begin{array}{cc}\frac{x y}{|x|+|y|}, & (x, y) \neq(0,0) \\ 0 & (x, y)=(0,0)\end{array}\right.$
(4) $f(x, y, z)=\left\{\begin{array}{cc}\frac{x^{3}+y^{3}+z^{3}}{x^{2}+y^{2}+z^{2}}, & (x, y, z) \neq(0,0,0) \\ 0 & (x, y, z)=(0,0,0)\end{array}\right.$
(5) $f(x, y, z)=\left\{\begin{array}{cc}\frac{x z-y^{2}}{x^{2}+y^{2}+z^{2}}, & (x, y, z) \neq(0,0,0) \\ 0 & (x, y, z)=(0,0,0)\end{array}\right.$
(6) $f(x, y, z)=\left\{\begin{array}{cl}\frac{x z y^{2}}{x^{2}+y^{2}+z^{2}}, & (x, y, z) \neq(0,0,0) \\ 0 & (x, y, z)=(0,0,0)\end{array}\right.$

Discuss the continuity of the following functions on their domain
(1) $h(x, y)=e^{x^{2+5 x y+y^{2}}}$
(2) $h(x, y)=\sin \sqrt{y-4 x^{2}}$
(3) $h(x, y, z)=\ln \left(36-4 x^{2}-y^{2}-9 z^{2}\right)$
(4) $h(x, y, z)=\frac{x z}{\sqrt{x^{2}+y^{2}+z^{2}-1}}$

| 16.3 | $4,6,8,12,13,16,17,21,22,27,29,32,34,36,39,42,47$ |
| :---: | :---: |
|  | Use the definition to find $f_{x}$ and $f_{y}$ for the function $f(x, y)=3 x^{2}-2 x y+y^{2}$ |
|  | Let $f(x, y)=e^{x-y} \sin (x+y)$. Show that $\left(f_{x}\right)^{2}+\left(f_{y}\right)^{2}=\frac{2(f(x, y))^{2}}{\sin ^{2}(x+y)}$ |
| 16.4 | 2, 9, 11, 12, 16, 18, 20, 39, 41 |
|  | Use the differential to approximate the change in the function $w=f(x, y, z)=x^{2} \ln \left(y^{2}+z^{2}\right)$ as ( $x, y, z$ ) changes from $(1,2,3)$ to $(0.9,1.9,3.1)$ |
|  | Use the differential to approximate the change in the function $w=f(x, y)=y x^{\frac{2}{5}}+\sqrt{x-y}$ as $(x, y)$ changes from $(52,16)$ to $(35,18)$ |
|  | Let $f(x, y, z)=\left\{\begin{array}{cc}\frac{x y^{2} z}{x^{4}+y^{4}+z^{4}}, & (x, y, z) \neq(0,0,0) \\ 0 & (x, y, z)=(0,0,0)\end{array}\right.$ <br> (a) Show that $f_{x}(0,0,0), f_{y}(0,0,0)$ and $f_{z}(0,0,0)$ exist <br> (b) Discuss the differentiability of $f(x, y, z)$ at $(0,0,0)$ |
| 16.5 | 2, 4, 6, 10, 12, 14, 18, 19, 22, 33, 37, 38, 41, 42 |
|  | Let $w=f(x, y, z)=x^{2}+y^{2}+z^{2}$, where $x=r \cos \theta, y=r \sin \theta$ and $z=r$. Use the differentials to show that $d w=4 r d r$ |
|  | Let $z=f(x, y)$ be determined implicitly by $x^{2}+z^{2}+\cos (x y z)-4=0$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, then show that $2 y \frac{\partial z}{\partial y}-x \frac{\partial z}{\partial x}=\frac{x y z \sin (x y z)}{2 z-x y \sin (x y z)}$ |
| 16.8 | 1, 9, 11, 15, 20, 21, 23, 24, 26, 29, 31, 32 |
| 16.9 | 1, 2, 3, 11 |
| 17.1 | $1,2,4,7,10,13,16,18,19,20,21,23,25,26,27,29,31,32,33,37,38,39,43,44,50$ |
|  | Sketch the region $R$ bounded by the graphs of $y=x, y=\sqrt{x}$ and $x=0$ then evaluate the integral $\iint_{R} \sin y^{2} d A$ |
|  | Evaluate the double integral $\int_{0}^{2} \int_{y / 2}^{1} e^{x^{2}} d x d y$ |
| 17.2 | $2,4,6,7,11,14,18,22,24,27,28,30,31,32$ |
|  | Sketch the region bounded by the graphs of the equations $y=\sin x, y=\cos x, x=0$ and $x=\frac{\pi}{4}$ then use a double integral to find its area. |
| 17.3 | $1,2,3,7,8,9,10,12,13,15,17,18,19,21,23,24$ |
|  | Use polar coordinates to evaluate the double integral $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^{2}}}\left(x^{2}+y^{2}\right)^{\frac{3}{2}} d y d x$ |
| 17.5 | $2,6,7,8,9,11,12,13,14,16,17,23,26,28$ |
| 17.7 | 2, 6, 7, 13, 18, 20, 23, 30(a), 40 |
| 17.8 | 2, 3, 11, 14, 25, 28, 35, 36, 39, 40 |


| 11.1 | $3,5,7,11,12,13,16,17,18,20,22,23,24,27-38,41,42$ |
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| 11.2 | $2,4,5,6,8,10,14,15,18,20,25,28,30,34,37,38,39,40,42,43,45,46,50,57,58$ |
| 11.3 | $2-10,14,15,16,18,20,22,23,24,25,26,30,31,33,34,35,39,40,42,43,45,46,57,58$ |
| 11.4 | $2,4,6,8,9,10,11,12,14,15,16,18,20,21,22,23,25,27,28,29,31,32,33,34,35,38$ |
| 11.5 | $2-7,9,10,12,13,14,16,19,20,21,22,27,28,29,32,43,44,45,46$ |
| 11.6 | $5,6,7,8,14,15,19,23,25,27,3035,36,41,42,44,45,46$ |
| 11.7 | $2,4,6,7,10,13,14,16,19,22,25,29,30,32,33,34,37$ |
| 11.8 | $2,4,8,10,13,15,18,19,21,26$ |

