# SECOND MID TERM EXAMINATION, SEM. II, 2025
## DEPT. MATH., COLLEGE OF SCIENCE
## KING SAUD UNIVERSITY
MATH: 107 FULL MARK: 25 TIME: 90 MIN.

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**Q1.**[2+2+2=6]

(a) A constant force $\mathbf{F} = \langle 5, -3, 1\rangle$ moves a body from point $P(1, 1, 1)$ to point $Q(9, 4, 7)$ along a straight line. Find the work done.

(b) Consider the vectors $\mathbf{a} = \langle x, 0, 0\rangle$, $\mathbf{b} = \langle 0, y, 0\rangle$ and $\mathbf{c} = \langle 0, 0, z\rangle$ with $\mathbf{a}+2\mathbf{b}+3\mathbf{c}=\langle 1, 2, 3\rangle$. Find values of $x$, $y$ and $z$.

(c) Show that the vectors **a**, **b** and **c** are mutually orthogonal.

**Q2.** [3+2+2+2=9]

(a) Find an equation of the plane through the points $P(1, -2, 0)$, $Q(2, 0, 3)$ and $R(0, -2, -3)$.

(b) Let $\wp_1$ and $\wp_2$ be two planes defined by their equations:

$$\wp_1 : x - 2y + 2z = 3$$
$$\wp_2 : 2x + y - z = 1$$

(i) Prove that $\wp_1$ and $\wp_2$ are not parallel.

(ii) Find parametric equations of the line of intersection of the planes $\wp_1$ and $\wp_2$.

(iii) Find the distance between the point $A(1, -1, 3)$ and the plane $\wp_1$.

**Q3.** [3+4+3=10]

(a) Let $\mathbf{r}(t) = \ln(1-t)\mathbf{i} + \sin t\mathbf{j} + t^2\mathbf{k}$. Find the domain of $\mathbf{r}$. Also, find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

(b) If $\mathbf{r}(t) = e^t(\cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k})$ is the position vector of a moving point $P$, find its velocity, acceleration, and speed at $t = \frac{\pi}{2}$.

(c) Sketch the graph of the surface $9(x^2 + z^2) + 4y^2 = 36$ in an $xyz$ coordinate system, describe the traces on the coordinate system, and identify the surface.

# SOLUTION MIDTERM EXAM. II, SEMESTER II, 2025

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**Q1.** [2+2+2=6] (a) The displacement vector $\mathbf{PQ} = \langle 8,3,6\rangle$. Hence, work done $= \mathbf{F}\cdot\mathbf{PQ} = \langle 5,-3,1\rangle\cdot\langle 8,3,6\rangle = 37$ units.

(b) $\langle 1,2,3\rangle = \mathbf{a}+2\mathbf{b}+3\mathbf{c} = \langle x,0,0\rangle + 2\langle 0,y,0\rangle + 3\langle 0,0,z\rangle = \langle x,2y,3z\rangle \implies x=y=z=1$.

(c) $\mathbf{a}\cdot\mathbf{b} = \langle 1,0,0\rangle\cdot\langle 0,1,0\rangle = 0$. Similarly, $\mathbf{b}\cdot\mathbf{c}=0$ and $\mathbf{c}\cdot\mathbf{a}=0$.

**Q2.** [3+2+2+2=9] (a) Let $\mathbf{a} = \mathbf{PQ} = \langle 1,2,3\rangle$ and $\mathbf{b} = \mathbf{PR} = \langle -1,0,-3\rangle$. Then $\mathbf{a}\times\mathbf{b} = \langle -6,0,2\rangle$. So the plane contains $P(1,-2,0)$ and has normal vector $\mathbf{a}\times\mathbf{b} = \langle -6,0,2\rangle$. This plane, then, has equation $-6(x-1)+2(z-0)=0$, that is, $-6x+2z+6=0$.

(b) (i) We have normal vectors $\mathbf{n}_1 = \langle 1,-2,2\rangle$ and $\mathbf{n}_2 = \langle 2,1,-1\rangle$ show that $\frac{1}{2} \neq \frac{-2}{1}$, then $p_1 \nparallel p_2$.

(ii) Considering the augmented matrix, we have

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 2 & 1 & -1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

where, we have $x = 3+2y-2z$ and $y = -1+z$. Using $z=t$, we get $x=1$, $y=-1+t$ and $z=t$, for all $t\in\Re$.

(iii) The distance from the point $A(1,-1,3)$ to the plane is $d = \frac{|1-2(-1)+2(3)-3|}{\sqrt{1^2+(-2)^2+2^2}} = \frac{6}{\sqrt{9}} = 2 \Rightarrow d = 2$.

**Q3.** [3+4+3=10] (a) Domain of $\mathbf{r}$ is $(-\infty,1)$. $\mathbf{r}'(t) = \frac{-1}{1-t}\mathbf{i}+\cos t\mathbf{j}+2t\mathbf{k}$. $\mathbf{r}''(t) = \frac{-1}{(1-t)^2}\mathbf{i}-\sin t\mathbf{j}+2\mathbf{k}$.

(b) $\mathbf{r}(t) = e^t\cos t\mathbf{i} + e^t\sin t + e^t\mathbf{k}$

$\mathbf{v}(t) = \mathbf{r}'(t) = (e^t\cos t - e^t\sin t)\mathbf{i} + (e^t\cos t + e^t\sin t)\mathbf{j} + e^t\mathbf{k}$

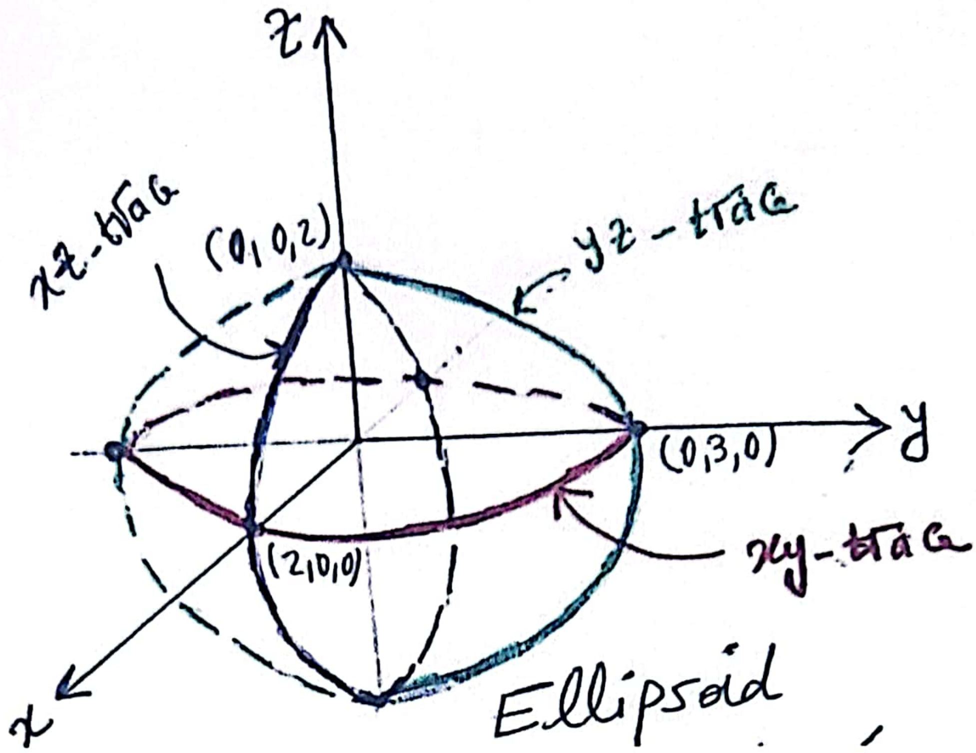
$\mathbf{a}(t) = \mathbf{r}''(t) = -2e^t\sin t\mathbf{i} + 2e^t\cos t\mathbf{j} + e^t\mathbf{k}$.

$\mathbf{v}(\frac{\pi}{2}) = -e^{\frac{\pi}{2}}\mathbf{i} + e^{\frac{\pi}{2}}\mathbf{j} + e^{\frac{\pi}{2}}\mathbf{k} = e^{\frac{\pi}{2}}(-\mathbf{i}+\mathbf{j}+\mathbf{k})$

$\mathbf{a}(\frac{\pi}{2}) = -2e^{\frac{\pi}{2}}\mathbf{i} + e^{\frac{\pi}{2}}\mathbf{k} = e^{\frac{\pi}{2}}(-2\mathbf{i}+\mathbf{k})$.

Speed$=\sqrt{e^{\pi}+e^{\pi}+e^{\pi}} = \sqrt{3e^{\pi}} = \sqrt{3}e^{\frac{\pi}{2}}$.

(c) The given equation of the surface can be written as $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{4}=1$, which is an ellipsoid. $xy$-trace is $\frac{x^2}{4}+\frac{y^2}{9}=1$, which is an ellipse; $yz$-trace is $\frac{y^2}{9}+\frac{z^2}{4}=1$, which is an ellipse; $xz$-trace : $\frac{x^2}{4}+\frac{z^2}{4}=1$, which is a circle. Sketch is given at the last page.


z
xz-trace
(0,0,2)
yz-trace
y
(0,3,0)
xy-trace
(2,0,0)
x
Ellipsoid