MATH 343 - GROUP THEORY

Course Syllabus and 15-Week Course Plan

First Semester 2026-2027 (14481)

Institution: King Saud University, College of Science, Department of Mathematics

Course: MATH 343 - Group Theory | 4 credit hours (3 lecture + 2 tutorial contact hours per week)

Sections: Lecture 4084; Tutorial 4087

Meeting pattern: Lectures Sunday, Tuesday, and Thursday, 10:00-10:50, Room A78; tutorial Monday, 10:00-11:50, Room A107

Instructor: Fahd M. Alshammari | Phone 4676521 | fmobarak@ksu.edu.sa

Prerequisites: MATH 246 (Linear Algebra) and MATH 243 (Number Theory); no co-requisite

Required textbook: Joseph A. Gallian, Contemporary Abstract Algebra, 10th ed., 2021

Course Description

This course develops the fundamental concepts and structures of group theory, including groups and subgroups, cyclic and permutation groups, isomorphisms and homomorphisms, cosets and quotient groups, direct products, finite Abelian groups, Sylow theory, and finite simple groups. Emphasis is placed on abstract reasoning, proof construction, structural analysis, and the use of symmetry and permutations as central examples.

Course Learning Outcomes

1.1 Define and explain the core concepts of group theory, including groups, subgroups, cyclic groups, cosets, and normal subgroups.

1.2 Describe and interpret key theorems such as Lagrange's Theorem and the Isomorphism Theorems, explaining their mathematical significance.

2.1 Apply definitions and theorems to analyze group structures, determine subgroup properties, and construct quotient groups.

2.2 Compute and verify algebraic properties such as orders, kernels, images, and isomorphisms to evaluate and classify group structures.

2.3 Prove results in group theory using appropriate logical and algebraic reasoning methods.

3.1 Demonstrate the ability to study, learn, and work independently for a continued professional growth in abstract algebra.

3.2 Collaborate effectively with peers to engage in group problem-solving and learning activities.

Course Scope

- Chapters 0 -11: introduction to groups; groups and subgroups; cyclic and permutation groups; isomorphisms; cosets and Lagrange's Theorem; external direct products; normal subgroups, factor groups, and internal direct products; homomorphisms; finite Abelian groups.
- Chapter 23: conjugacy classes, the class equation, Sylow Theorems, and applications.
- Chapter 24: nonsimplicity tests and the simplicity of A_5.

Assessment and Grading

Component	Weight	Timing / details
Assignments	10%	TBD
Quizzes	10%	TBD
Midterm 1	20%	Monday Oct 5, 2026 (10:00 – 11:30)
Midterm 2	20%	Monday Nov 9, 2026 (10:00 – 11:30)
Final examination	40%	Sunday Dec 20, 2026 (8:00 – 11:00)

15-Week Tentative Course Plan

Wk.	Textbook sections	Lecture and tutorial emphasis
1	Ch. 0 (selected); Ch. 1; Ch. 2: Definition and Examples of Groups	Brief prerequisite check (integers, modular arithmetic, functions); symmetries of a square and dihedral groups; group axioms and standard examples. Tutorial: identify examples/non-examples and verify group axioms.
2	Ch. 2: Elementary Properties; Ch. 3: Terminology, Subgroup Tests, Examples	Identity and inverse properties; order; subgroup notation and tests; centers, centralizers, and generated subgroups. Tutorial: subgroup-test proofs and counterexamples.
3	Ch. 4: Properties of Cyclic Groups; Classification of Subgroups	Generators and element orders; structure and subgroups of cyclic groups; Euler phi applications. Tutorial: computations followed by proof problems.
4	Ch. 5: Definitions and Notation; Cycle Notation; Properties of Permutations	Permutation composition convention; cycles, disjoint cycles, inverses, transpositions, parity, Sn and An within announced scope. Tutorial: permutation calculations and short proofs.
5	Ch. 6: Motivation; Definition and Examples; Properties of Isomorphisms	Construct and test isomorphisms; invariants; recognize nonisomorphic groups. Tutorial: candidate maps, preservation arguments, and structural comparisons.
6	Ch. 6: Automorphisms; Cayley's Theorem; Ch. 7: Properties of Cosets	Automorphism groups and inner automorphisms; Cayley representation; left/right cosets and representatives. Tutorial: automorphisms, Cayley embeddings, and coset computations.
7	Ch. 7: Lagrange's Theorem and Consequences	Index, element orders, subgroup-order restrictions, and standard consequences; selected permutation-group applications. Tutorial: theorem selection, hypothesis checks, and converse counterexamples.
8	Ch. 8: Definition and Examples; Properties; Units Modulo n	External direct products; element orders and cyclicity; decomposing U(n) where appropriate. Tutorial: product computations and classification examples.
9	Ch. 9: Normal Subgroups; Factor Groups; Applications	Normality criteria, conjugation, quotient operations, and well-definedness. Tutorial: normality tests, coset multiplication, and quotient examples.
10	Ch. 9: Internal Direct Products; Ch. 10: Homomorphisms	Internal versus external products; definition and examples of homomorphisms; kernels, images, and basic properties. Tutorial: verify maps; compute kernels and images.

Wk.	Textbook sections	Lecture and tutorial emphasis
11	Ch. 10: First Isomorphism Theorem; Ch. 11: Fundamental Theorem	First Isomorphism Theorem and applications; statement and use of the finite Abelian group classification theorem. Tutorial: quotient-image isomorphisms and classification tables.
12	Ch. 11: Isomorphism Classes; Ch. 23: Conjugacy Classes and Class Equation	Classify finite Abelian groups by order; conjugation, centers, centralizers, class sizes, and class equation. Tutorial: decomposition and class-equation computations.
13	Ch. 23: Sylow Theorems	Existence, conjugacy, and counting statements; immediate structural consequences. Tutorial: determine possible Sylow numbers and prove existence/normality conclusions.
14	Ch. 23: Applications; Ch. 24: Nonsimplicity Tests	Combine Sylow theory, conjugation actions, and index arguments to analyze finite groups; first nonsimplicity tests. Tutorial: structured proof workshop.
15	Ch. 24: Simplicity of A_5; Comprehensive Synthesis	Proof architecture for simplicity of A_5 at the course level; connections among subgroups, quotients, homomorphisms, products, and Sylow theory. Tutorial: final review and mixed problems.

Suggested Textbook Exercises by Chapter

Chapter 2. Groups: 5, 10, 13, 14, 16, 21, 27, 36, 42, 45

Chapter 3. Finite Groups; Subgroups: 1, 17, 21, 27, 38, 55, 67, 71

Chapter 4. Cyclic Groups: 1, 8, 9, 23, 24, 31, 61

Chapter 5. Permutation Groups: 2, 5, 8, 27, 46, 47

Chapter 6. Isomorphisms: 2, 4, 6, 7, 8, 12, 17, 31, 41

Chapter 7. Cosets and Lagrange's Theorem: 1, 4, 13, 16, 20, 24, 35, 59

Chapter 8. External Direct Products: 1, 4, 6, 11, 17, 31, 50, 62

Chapter 9. Normal Subgroups and Factor Groups: 2, 9, 10, 11, 13, 32

Chapter 10. Group Homomorphisms: 8, 9, 12, 24

Chapter 11. Fundamental Theorem of Finite Abelian Groups: 2, 4, 5, 9, 10, 12, 15, 21, 28, 42

Chapter 23. Sylow Theorems: 1, 6, 13, 14, 22, 24, 25

Chapter 24. Finite Simple Groups: 1, 12, 14, 17, 18, 20, 21, 23

Course Policies and Expectations

- Attendance: A student may not miss more than 25% of the total lectures and tutorials. Exceeding this threshold may result in denial of entry to the final examination and a DN grade, subject to university regulations.
- Punctuality: Attendance is recorded at the start of class. More than two arrivals over five minutes late may be counted as one absence.
- Missed examinations: A missed midterm requires acceptable documentation, such as a medical report or official leave, and approval through the authorized departmental process.

- Assignments integrity: Discussion with classmates is encouraged, but submitted work must be written independently. Direct copying is academic misconduct and may receive a grade of zero under course and university rules.
- Classroom conduct: Keep mobile phones silent and put away, avoid side conversations, and raise questions freely and respectfully.

Wishing you a successful and enjoyable course.