

**Question 1 :**

a)  $F'(x) = \cosh(\sin x) \cdot \cos x - 2x \cosh(x^2), \quad (1)$

$F'(0) = 1 \quad (1).$

b)  $\int_1^2 \frac{dx}{x^2} = \frac{1}{2}. \quad (1)$

$\frac{1}{c^2} = \frac{1}{2} \Rightarrow c = \sqrt{2}$  since  $-\sqrt{2} \notin [1, 2]. \quad (1)$

**Question 2 :**

a)

$$\int \frac{dx}{\sqrt{3^{2x} - 1}} \stackrel{u=3^x}{=} \frac{1}{\ln 3} \int \frac{du}{u\sqrt{u^2 - 1}}. \quad (2)$$

$$= \frac{1}{\ln 3} \sec^{-1}(3^x) + c. \quad (1)$$

b)

$$\int \frac{\cot x}{\sqrt{1 - \sin^2 x}} dx \stackrel{u=\sin x}{=} \int \frac{du}{u\sqrt{1 - u^2}}. \quad (2)$$

$$= -\operatorname{sech}^{-1}(|\sin x|) + c. \quad (1)$$

c)  $Y = x^{\sin x}, \ln Y = \sin x \ln x$

$$\begin{aligned} \lim_{x \rightarrow 0^+} \ln Y &= \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{\sin x}} \\ &= \lim_{x \rightarrow 0^+} \frac{-\sin^2 x}{x \cos x} = 0. \quad (2.5) \end{aligned}$$

$$\lim_{x \rightarrow 0^+} x^{\sin x} = 1. \quad (0.5)$$

Question 3 :

a)

$$\begin{aligned}
 \int x \tan^{-1} x dx &= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx & (1.5) \\
 &= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int 1 - \frac{1}{1+x^2} dx \\
 &= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + c. & (1.5)
 \end{aligned}$$

b)

$$\begin{aligned}
 \int \frac{dx}{(1+x^2)^2} &\stackrel{x=\tan\theta}{=} \int \frac{\sec^2\theta}{\sec^4\theta} d\theta \\
 &= \int \frac{1}{2} + \frac{1}{2} \cos(2\theta) d\theta \\
 &= \frac{\theta}{2} + \frac{1}{2} \sin\theta \cos\theta + c & (2) \\
 &= \frac{1}{2} \left( \tan^{-1} x + \frac{x}{1+x^2} \right) + c. & (1)
 \end{aligned}$$

c)

$$\begin{aligned}
 \int \frac{dx}{\sqrt{x^2 + 8x + 25}} &\stackrel{u=x+4}{=} \int \frac{du}{\sqrt{9+u^2}} & (2) \\
 &= \sinh^{-1} \left( \frac{x+4}{3} \right) + c. & (1)
 \end{aligned}$$

Question 4 :

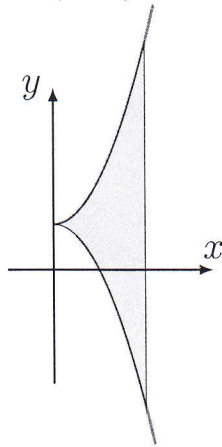
a)

$$\begin{aligned} \int \frac{dx}{x^{\frac{1}{4}} + x^{\frac{1}{2}}} &\stackrel{u=x^{\frac{1}{4}}}{=} 4 \int \frac{u^3}{u + u^2} du \\ &= 4 \int \left(u - 1 + \frac{1}{1+u}\right) du \quad (2) \\ &= 2x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 4 \ln(1 + x^{\frac{1}{4}}) + c. \quad (1) \end{aligned}$$

$$\text{b) } \int_2^c \frac{dx}{x(\ln x)^3} \stackrel{u=\ln x}{=} \int_{\ln 2}^{\ln c} \frac{du}{u^3} = -\frac{1}{2} \left[ \frac{1}{u^2} \right]_{\ln 2}^{\ln c} \quad (1.5)$$

$\lim_{c \rightarrow +\infty} \int_2^c \frac{dx}{x(\ln x)^3} = \frac{1}{2(\ln 2)^2}$ . So the integral converges and

$$\int_2^{+\infty} \frac{dx}{x(\ln x)^3} = \frac{1}{2(\ln 2)^2}. \quad (1.5)$$



c) (1)

Intersection points  $x^2 + 1 = 1 - x^2 \Rightarrow x = 0$ .

$$A = \int_0^2 2x^2 dx = \frac{16}{3}. \quad (2)$$

Question 5 :

a)  $4 - x^2 = 0 \iff x = \pm 2.$

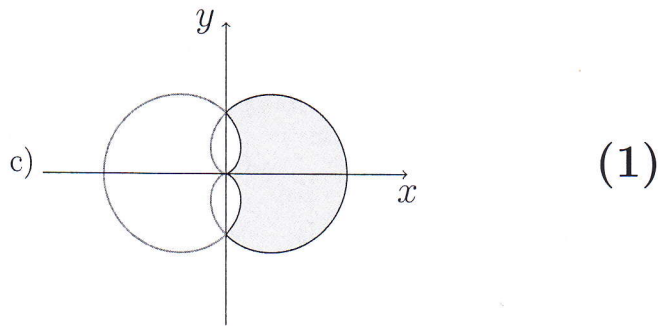
$$V = \int_0^2 2\pi x(4 - x^2) dx \quad (2)$$

$$= 2\pi \left[ 2x^2 - \frac{1}{4}x^4 \right]_0^2 = 8\pi. \quad (1)$$

b)

$$S = 2\pi \int_0^1 t^3 \sqrt{4 + 9t^4} dt \quad (1.5)$$

$$= \frac{\pi}{18} \int_4^{13} \sqrt{u} du = \frac{\pi}{27} (13^{\frac{3}{2}} - 8). \quad (1.5)$$



$$A = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + \cos \theta)^2 - (1 - \cos \theta)^2 d\theta \quad (1)$$

$$= \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4 \cos \theta d\theta = 4. \quad (1)$$