

111 رياض - حساب التكامل  
 الفصل الدراسي الأول 1444 هـ  
 حل الاختبار الفصلي  
 د. طارق عبدالرحمن محمد الفاضل

السؤال الأول (12 درجة) :

$$(1) \text{ استخدم مجموع ريمان لحساب التكامل المحدد } \int_{-1}^4 (2x+1) dx$$

الحل :  $f(x) = 2x + 1$  و  $[a, b] = [-1, 4]$

$$\Delta_x = \frac{b-a}{n} = \frac{4-(-1)}{n} = \frac{4+1}{n} = \frac{5}{n}$$

$$x_k = a + k \Delta_x = -1 + k \left( \frac{5}{n} \right) = -1 + \frac{5k}{n}$$

$$f(x_k) = 2 \left( -1 + \frac{5k}{n} \right) + 1 = -2 + \frac{10k}{n} + 1 = \frac{10k}{n} - 1$$

$$R_n = \sum_{k=1}^n f(x_k) \Delta_x = \sum_{k=1}^n \left( \frac{10k}{n} - 1 \right) \left( \frac{5}{n} \right) = \sum_{k=1}^n \left( \frac{50k}{n^2} - \frac{5}{n} \right) = \sum_{k=1}^n \frac{50k}{n^2} - \sum_{k=1}^n \frac{5}{n}$$

$$= \frac{50}{n^2} \sum_{k=1}^n k - \frac{5}{n} \sum_{k=1}^n 1 = \frac{50}{n^2} \frac{n(n+1)}{2} - \frac{5}{n} (n) = 25 \left( \frac{n+1}{n} \right) - 5$$

$$\int_{-1}^4 (2x+1) dx = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \left[ 25 \left( \frac{n+1}{n} \right) - 5 \right] = 25(1) - 5 = 20$$

$$(2) \text{ جد } F'(x) \text{ إذا كانت } F(x) = \int_{\sqrt{3x}}^{\ln 2x} \frac{1}{\sqrt{t^2-2t}} dt$$

الحل :

$$F'(x) = \frac{d}{dx} \int_{\sqrt{3x}}^{\ln 2x} \frac{1}{\sqrt{t^2-2t}} dt$$

$$= \frac{1}{\sqrt{(\ln 2x)^2 - 2 \ln 2x}} \left( \frac{2}{2x} \right) - \frac{1}{\sqrt{(\sqrt{3x})^2 - 2\sqrt{3x}}} \left( \frac{1}{2\sqrt{3x}} (3) \right)$$

$$= \frac{1}{x \sqrt{(\ln 2x)^2 - 2 \ln 2x}} - \frac{3}{2\sqrt{3x} \sqrt{3x - 2\sqrt{3x}}}$$

احسب  $\frac{dy}{dx}$  فيما يلي :

$$y = 7^{x^2} \cosh^{-1}(x^2) \quad (3)$$

: الحل

$$\begin{aligned} \frac{dy}{dx} &= \left(7^{x^2} (2x) \ln 7\right) \cosh^{-1}(x^2) + 7^{x^2} \left[ \frac{1}{\sqrt{(x^2)^2 - 1}} (2x) \right] \\ &= 2x 7^{x^2} \ln 7 \cosh^{-1}(x^2) + \frac{2x 7^{x^2}}{\sqrt{x^4 - 1}} \end{aligned}$$

$$y = \sin^{-1}(2x) \log_5 |3 - \tan 6x| \quad (4)$$

: الحل

$$\begin{aligned} \frac{dy}{dx} &= \left( \frac{1}{\sqrt{1 - (2x)^2}} (2) \right) \log_5 |3 - \tan 6x| + \sin^{-1}(2x) \left( \frac{-\sec^2(6x) (6)}{3 - \tan 6x} \frac{1}{\ln 5} \right) \\ &= \frac{2 \log_5 |3 - \tan 6x|}{\sqrt{1 - 4x^2}} - \frac{6 \sec^2(6x) \sin^{-1}(2x)}{(3 - \tan 6x) \ln 5} \end{aligned}$$

$$y = (\tan x)^{\sin x} \quad (5)$$

: الحل

$$y = (\tan x)^{\sin x} \implies \ln |y| = \ln |(\tan x)^{\sin x}| = \sin x \ln |\tan x|$$

ياشتقاق الطرفين

$$\frac{y'}{y} = (\cos x) \ln |\tan x| + \sin x \left( \frac{\sec^2 x}{\tan x} \right)$$

$$y' = y \left[ \cos x \ln |\tan x| + \frac{\sin x \sec^2 x}{\tan x} \right]$$

$$y' = (\tan x)^{\sin x} [\cos x \ln |\tan x| + \sec x]$$

$$y = \coth^{-1} \left( \frac{1}{x} \right) \quad (6)$$

: الحل

$$\frac{dy}{dx} = \frac{-1}{1 - \left(\frac{1}{x}\right)^2} \left( \frac{-1}{x^2} \right) = \frac{1}{x^2 \left[1 - \frac{1}{x^2}\right]} = \frac{1}{x^2 - 1}$$

السؤال الثاني (18 درجة): أحسب التكاملات التالية

$$\int \frac{\csc^2(e^{2x})}{e^{-2x}} dx \quad (1)$$

الحل :

$$\begin{aligned} \int \frac{\csc^2(e^{2x})}{e^{-2x}} dx &= \int \csc^2(e^{2x}) e^{2x} dx = \frac{1}{2} \int \csc^2(e^{2x}) e^{2x} (2) dx \\ &= \frac{1}{2} (-\cot(e^{2x})) + c = -\frac{1}{2} \cot(e^{2x}) + c \end{aligned}$$

باستخدام القانون

$$\int \csc^2(f(x)) f'(x) dx = -\cot(f(x)) + c$$

$$\int (x^3 + 4)^3 x^2 dx \quad (2)$$

الحل :

$$\int (x^3 + 4)^3 x^2 dx = \frac{1}{3} \int (x^3 + 4)^3 (3x^2) dx = \frac{1}{3} \frac{(x^3 + 4)^4}{4} + c$$

$$\text{باستخدام القانون } \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c \text{ حيث } n \neq -1$$

$$\int_0^1 x 10^{2-x^2} dx \quad (3)$$

الحل :

$$\begin{aligned} \int_0^1 x 10^{2-x^2} dx &= \frac{1}{-2} \int_0^1 10^{2-x^2} (-2x) dx = \frac{1}{-2} \left[ \frac{10^{2-x^2}}{\ln 10} \right]_0^1 \\ &= \frac{1}{-2} \left[ \frac{10^{2-(1)^2}}{\ln 10} - \frac{10^{2-(0)^2}}{\ln 10} \right] = \frac{1}{-2} \left[ \frac{10^1}{\ln 10} - \frac{10^2}{\ln 10} \right] = \frac{1}{-2} \left( \frac{-90}{\ln 10} \right) = \frac{45}{\ln 10} \\ &\text{باستخدام القانون } \int a^{f(x)} f'(x) dx = \frac{a^{f(x)}}{\ln a} + c \end{aligned}$$

$$\int \frac{x+2}{x+1} dx \quad (4)$$

الحل :

$$\begin{aligned} \int \frac{x+2}{x+1} dx &= \int \frac{(x+1)+1}{x+1} dx = \int \left( \frac{x+1}{x+1} + \frac{1}{x+1} \right) dx \\ &= \int \left( 1 + \frac{1}{x+1} \right) dx = \int 1 dx + \int \frac{1}{x+1} dx = x + \ln|x+1| + c \\ &\quad \cdot \int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c \text{ باستخدام القانون} \end{aligned}$$

$$\int \frac{e^{2x}}{\sqrt{4+e^{4x}}} dx \quad (5)$$

الحل :

$$\begin{aligned} \int \frac{e^{2x}}{\sqrt{4+e^{4x}}} dx &= \int \frac{e^{2x}}{\sqrt{(2)^2 + (e^{2x})^2}} dx = \frac{1}{2} \int \frac{2e^{2x}}{\sqrt{(2)^2 + (e^{2x})^2}} dx \\ &= \frac{1}{2} \sinh^{-1} \left( \frac{e^{2x}}{2} \right) + c \end{aligned}$$

باستخدام القانون

$$\int \frac{f'(x)}{\sqrt{a^2 + [f(x)]^2}} dx = \sinh^{-1} \left( \frac{f(x)}{a} \right) + c$$

$$\int \frac{\tanh(\sqrt{x})}{\sqrt{x}} dx \quad (6)$$

الحل :

$$\begin{aligned} \int \frac{\tanh(\sqrt{x})}{\sqrt{x}} dx &= \int \tanh(\sqrt{x}) \left( \frac{1}{\sqrt{x}} \right) dx = 2 \int \tanh(\sqrt{x}) \left( \frac{1}{2\sqrt{x}} \right) dx \\ &= 2 \ln |\cosh(\sqrt{x})| + c \end{aligned}$$

باستخدام القانون

$$\int \tanh(f(x)) f'(x) dx = \ln |\cosh(f(x))| + c$$

$$\int \frac{3}{9x^2+4} dx \quad (7)$$

الحل :

$$\int \frac{3}{9x^2+4} dx = \int \frac{3}{(3x)^2 + (2)^2} dx = \frac{1}{2} \tan^{-1} \left( \frac{3x}{2} \right) + c$$

باستخدام القانون

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{f(x)}{a} \right) + c$$

$$\int (x+3) \sqrt{x+2} dx \quad (8)$$

الحل : بوضع  $u = x + 2 \implies u + 1 = x + 3$

$$dx = du$$

$$\begin{aligned} \int (x+3) \sqrt{x+2} dx &= \int (u+1) \sqrt{u} du = \int (u+1) u^{\frac{1}{2}} du \\ &= \int \left( u^{\frac{3}{2}} + u^{\frac{1}{2}} \right) du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{5} (x+2)^{\frac{5}{2}} + \frac{2}{3} (x+2)^{\frac{3}{2}} + c \end{aligned}$$

$$\int e^{3x} \cosh x dx \quad (9)$$

الحل :

$$\begin{aligned} \int e^{3x} \cosh x dx &= \int e^{3x} \left( \frac{e^x + e^{-x}}{2} \right) dx = \int \left( \frac{e^{4x} + e^{2x}}{2} \right) dx \\ &= \int \left( \frac{e^{4x}}{2} + \frac{e^{2x}}{2} \right) dx = \frac{1}{2} \frac{1}{4} \int e^{4x} (4) dx + \frac{1}{2} \frac{1}{2} \int e^{2x} (2) dx \\ &= \frac{1}{8} e^{4x} + \frac{1}{4} e^{2x} + c = \frac{e^{4x}}{8} + \frac{e^{2x}}{4} + c \end{aligned}$$

$$\int e^{f(x)} f'(x) dx = e^{f(x)} + c \quad \text{باستخدام القانون}$$