

List of Exercises:

Chapter 1.

[Parabola]

Find the elements of the parabola and sketch it in the exercises 1-9:

1. $y^2 = 3x$

2. $x^2 = -2y$

3. $x^2 = 4y$

4. $x^2 = -2y - 1$

5. $(x + 1)^2 = -y - 3$

6. $(y - 4)^2 = -8(x + 2)$

7. $y^2 = 4y + 2x + 4$

8. $x^2 - 4x = y$

9. $x^2 + 8y + 6x + 1 = 0$

Find the equation of the parabola and sketch it in the exercises 10-18:

10. The vertex V is the origin and the focus F is $(3,0)$.

11. The vertex V is the origin and the equation of the directrix is $x=-5$.

12. The vertex V is the origin and the equation of the directrix is $y=2$.

13. The vertex V is $(1,2)$ and the focus F is $(3,2)$.

14. The vertex V is $(-1,1)$ and the equation of the directrix is $x=2$.

15. The focus F is $(3,1)$ and the equation of the directrix is $y=-3$.

16. The focus F is $(1,2)$ and the equation of the directrix is $y=1$.

17. The vertex V is $(0.5,-1.5)$ and the equation of the directrix is $x=-1$

18. The vertex V is $(1,-1)$ and the focus F is $(1,3)$.

[Ellipse]

Find the elements of the ellipse and sketch it in the exercises 19-22:

19. $9y^2 + 25x^2 = 225$

20. $9y^2 + 4x^2 = 36$

21. $9y^2 + 25x^2 - 18y - 100x = 116$

22. $9y^2 + 4x^2 + 18y - 8x = 23$

Find the equation of the ellipse and sketch it in the exercises 23-26:

23. The foci are $F_1(2, 0)$ and $F_2(-2, 0)$, and it passes through the point $A(0, 1)$.

24. The vertices are $V_1(3, 0)$ and $V_2(-3, 0)$, and the length of the minor axis is 4.

25. The foci are $F_1(2, 3)$ and $F_2(-2, 3)$, and it passes through the point $A(0, 4)$.

26. The vertices are $V_2(-3, 2)$ and $V_1(3, 2)$, and the length of the minor axis is 4.

[Hyperbola]

Find the elements of the hyperbola and sketch it in the exercises 27-30:

27. $9y^2 - 25x^2 = 225$

28. $9y^2 - 16x^2 = 144$

29. $9y^2 - 25x^2 - 18y + 100x = 316$

30. $4y^2 - 8y - 9x^2 - 18x = 31$

Find the equation of the hyperbola and sketch it in the exercises 23-26:

31. The foci are $F_1(2, 0)$ and $F_2(-2, 0)$, and has one vertex on $(1, 0)$.

32. The vertices are $V_1(3, 0)$ and $V_2(-3, 0)$, and has one focus on $(4, 0)$.

33. The foci are $F_1(5, 4)$ and $F_2(-5, 4)$, and the distance between the vertices is 8.

34. The vertices are $V_1(2, 4)$ and $V_2(2, -4)$, and the foci are $F_1(2, 5)$ and $F_2(2, -5)$

[Miscellaneous]

35. Find the elements of the conic section: $y = 2x^2 + 4x + 4$ and sketch it.
36. Find the equation of the parabola with focus $F(0,0)$ and its directrix with equation: $x = -2$.
37. Find the elements of the conic section: $4y^2 = -9x^2 + 18x + 27$ and sketch it.
38. Find the equation of the ellipse with foci $F_1(5,0)$ and $F_2(-5,0)$ and with vertices $V_1(8,0)$ and $V_2(-8,0)$.
39. Find the equation of the ellipse with foci $F_1(1,4)$ and $F_2(1,2)$ and the length of its major axis is 4.
40. Find the elements of the conic section: $4y^2 = 9x^2 + 18x + 45$ and sketch it.
41. Find the elements of the conic section: $x^2 - 2y^2 = 8$ and sketch it.
42. Find the equation of the hyperbola with foci $F_1(0,5)$ and $F_2(0,-5)$ and the distance between the vertices is 8.
43. Find the equation of the hyperbola with vertices $V_1(5,0)$ and $V_2(-5,0)$ and, with foci $F_1(8,0)$ and $F_2(-8,0)$.
44. Find the equation of the hyperbola with vertices $V_1(3,0)$ and $V_2(-3,0)$ and the equations of the asymptotes are $y = 2x$ and $y = -2x$.
45. Find the equation of the hyperbola with vertices $V_1(4,-4)$ and $V_2(4,4)$ and the distance between the foci is 10.

Chapter 2 . [Matrices and determinants]

Compute (if possible): $2BA$ and AB for the following matrices:

$$1) \quad A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 5 & -6 \\ 2 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ 0 & 4 \end{pmatrix};$$

$$2) \quad A = \begin{pmatrix} 3 & 7 \\ 2 & 2 \\ 1 & -4 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 1 & 2 \end{pmatrix};$$

$$3) \quad A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 0 & 2 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix};$$

$$4) \quad A = \begin{pmatrix} 0 & 2 & 0 \\ 3 & 2 & 3 \\ 0 & 2 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix};$$

Evaluate the following multiplications:

$$1) \quad (-4 \ 3 \ 8 \ -5) \begin{pmatrix} 7 \\ -6 \\ 0 \\ 3 \end{pmatrix} \quad (-3 \ 6 \ -9) \begin{pmatrix} 8 \\ -3 \\ 4 \end{pmatrix} \quad (5 \ 3 \ 7) \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix}$$

$$2) \quad (3 \ -1) \begin{pmatrix} 4 & 7 & -2 & 4 & 0 \\ 2 & -6 & 8 & -3 & 7 \end{pmatrix} \quad (4 \ -2) \begin{pmatrix} 3 & 6 \\ -1 & 5 \end{pmatrix}$$

$$3) \quad (-3 \ 6 \ -1) \begin{pmatrix} 5 & -2 & 7 \\ -6 & 2 & 5 \\ 9 & 0 & -1 \end{pmatrix} \quad (2 \ 8 \ 1) \begin{pmatrix} 5 & 6 \\ 3 & 1 \\ 1 & 0 \end{pmatrix}$$

$$4) \quad (9 \ 1 \ 4) \begin{pmatrix} 6 & 2 & 1 & 5 & -8 \\ 7 & 1 & -3 & 0 & 3 \\ 1 & -5 & 0 & 3 & 2 \end{pmatrix}$$

$$5) \quad (2 \ 4 \ 3 \ -1 \ 0) \begin{pmatrix} 4 & 3 & 5 & -1 & 0 \\ 1 & 3 & 5 & -2 & 6 \\ 2 & -4 & 3 & 6 & 5 \\ 3 & 4 & -1 & 6 & 5 \\ 2 & 3 & 1 & 4 & -6 \end{pmatrix}$$

$$6) \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -2 \\ 3 & 5 & 0 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 2 & 0 & 1 \\ 1 & 5 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$7) \begin{pmatrix} 7 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 4 & 1 & 2 \\ 3 & 2 & 5 \end{pmatrix} \quad \begin{pmatrix} 0 & 2 & -1 & 4 \\ 1 & 5 & 3 & 0 \\ -2 & 0 & 1 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ -1 \\ 3 \end{pmatrix}$$

$$8) \begin{pmatrix} 4 & 2 & -1 \\ -2 & 1 & 6 \end{pmatrix} \begin{pmatrix} 5 & -1 & 2 & 3 \\ 1 & 3 & 5 & 2 \\ 0 & 1 & -2 & 4 \end{pmatrix} \quad \begin{pmatrix} 3 & 1 & 4 \\ 1 & -3 & 5 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ -1 & 5 \\ 2 & -6 \end{pmatrix}$$

$$9) \begin{pmatrix} 5 & 1 \\ -2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} -4 & 1 \\ 0 & 5 \end{pmatrix} \quad \begin{pmatrix} 2 & 4 \\ 0 & -1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 0 & -2 & 3 & -2 \\ 1 & 3 & -4 & 1 \end{pmatrix}$$

$$10) \begin{pmatrix} 4 & 1 & -2 & 3 \\ 0 & 2 & 1 & 5 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 0 & 2 \\ 3 & 1 \\ 0 & -2 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 & 3 & 1 \\ 2 & 1 & 0 & -3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 0 \\ 3 & 0 & -1 & 2 \\ -1 & 4 & 3 & 0 \\ 2 & 0 & -2 & 1 \end{pmatrix}$$

$$11) \begin{pmatrix} 3 & 1 \\ -2 & 0 \\ 1 & -3 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & -2 \\ 2 & 4 & 1 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} 4 & 0 & 1 & -2 \\ 0 & 2 & 1 & 3 \\ 1 & 0 & -3 & 2 \end{pmatrix}$$

$$12) \begin{pmatrix} 2 & 1 & 0 & -1 \\ 1 & 2 & -1 & 1 \\ 0 & -2 & 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 3 & 1 & -1 & 3 \\ 2 & 1 & 0 & 3 \\ 3 & 5 & 1 & 0 \end{pmatrix}$$

Evaluate the following determinants:

$$1) \begin{vmatrix} 9 & 8 \\ 7 & 5 \end{vmatrix} \quad \begin{vmatrix} 3 & 2 \\ 1 & -4 \end{vmatrix} \quad \begin{vmatrix} -2 & 5 \\ 3 & -3 \end{vmatrix}$$

$$2) \begin{vmatrix} 0 & 5 \\ -3 & -4 \end{vmatrix} \quad \begin{vmatrix} 8 & 3 \\ -1 & 2 \end{vmatrix} \quad \begin{vmatrix} -4 & 5 \\ 7 & -2 \end{vmatrix}$$

$$3) \begin{vmatrix} 7 & -6 \\ -5 & 4 \end{vmatrix} \quad \begin{vmatrix} -\frac{2}{3} & \frac{2}{5} \\ \frac{1}{3} & \frac{4}{5} \end{vmatrix} \quad \begin{vmatrix} \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{4} \end{vmatrix}$$

$$4) \begin{vmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} \quad \begin{vmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \\ 3 & -2 & 4 \end{vmatrix} \quad \begin{vmatrix} -3 & 1 & 2 \\ 0 & -1 & 5 \\ 6 & 0 & 1 \end{vmatrix}$$

$$5) \begin{vmatrix} -1 & 0 & 3 \\ 2 & 0 & -2 \\ 1 & -3 & 4 \end{vmatrix} \quad \begin{vmatrix} 5 & 1 & 2 \\ -3 & 2 & -1 \\ 4 & -3 & 5 \end{vmatrix} \quad \begin{vmatrix} 1 & 0 & 3 \\ 4 & 4 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$6) \begin{vmatrix} 2 & 1 & 3 \\ 0 & -2 & 4 \\ 0 & 1 & 5 \end{vmatrix} \quad \begin{vmatrix} 1 & 5 & 4 \\ -3 & 6 & -2 \\ -1 & 5 & 3 \end{vmatrix} \quad \begin{vmatrix} 4 & 3 & 1 & 0 \\ -1 & 2 & -3 & 5 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -3 & 5 \end{vmatrix}$$

$$7) \begin{vmatrix} -1 & 3 & 0 & 2 \\ 2 & -1 & 1 & 0 \\ 5 & 2 & -2 & 0 \\ 1 & -1 & 3 & 1 \end{vmatrix} \quad \begin{vmatrix} 2 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 \\ 1 & 3 & 2 & 1 \\ 3 & 1 & 1 & -2 \end{vmatrix} \quad \begin{vmatrix} 1 & 2 & -1 & 1 \\ -1 & 1 & 2 & 3 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & -1 & 1 \end{vmatrix}$$

Chapter 3 . [Linear Systems]

Solve by one of the following methods (Cramer's Rule, Gauss elimination, and Gauss-Jordan Elimination) the following linear systems:

$$1) \begin{cases} x-2y+z=0 \\ -x+y=1 \\ 4x+z=7 \end{cases} ; \begin{cases} 4x+2y=3 \\ -4x+y=6 \end{cases} \quad \begin{cases} x-y=10 \\ x+y=4 \end{cases} \quad \begin{cases} 2x+y=11 \\ 3x-y=4 \end{cases}$$

$$2) \begin{cases} 4x-5y=-34 \\ 2x-3y=-22 \end{cases} \quad \begin{cases} 3x-2y=-15 \\ 5x+6y=3 \end{cases} \quad \begin{cases} 2x-3y=5 \\ 3x+3y=10 \end{cases}$$

$$3) \begin{cases} 7x-4y=81 \\ 5x-3y=57 \end{cases} \quad \begin{cases} 2x-3y=3 \\ 4x+5y=39 \end{cases} \quad \begin{cases} x+4y=11 \\ 5x-2y=11 \end{cases}$$

$$4) \begin{cases} 5x-2y=3 \\ 2x+3y=5 \end{cases} \quad \begin{cases} 3x-2y=1 \\ 2x+y=10 \end{cases} \quad \begin{cases} 3x+4y=85 \\ 5x+4y=107 \end{cases}$$

$$5) \begin{cases} x + y + z = 18 \\ x - y + z = 6 \\ x + y - z = 4 \end{cases} \quad \begin{cases} 2x + y = -3 \\ x + 3y = 19 \end{cases} \quad \begin{cases} 3x + y = 9 \\ x + 2y = 8 \end{cases}$$

$$6) \begin{cases} x + y + z = 35 \\ x - 2y + 3z = 15 \\ y - x + z = -5 \end{cases} \quad \begin{cases} x + y = 35 \\ x + z = 40 \\ y + z = 45 \end{cases} \quad \begin{cases} x + y + z = 12 \\ x - y = 2 \\ x - z = 4 \end{cases}$$

$$7) \begin{cases} x - 2y + 2z = 5 \\ 5x + 3y + 6z = 57 \\ x + 2y + 2z = 21 \end{cases} \quad \begin{cases} x + y + z = 90 \\ 2x - 3y = -20 \\ 2x + 3z = 145 \end{cases} \quad \begin{cases} x + 2y + 3z = 14 \\ 2x + y + 2z = 10 \\ 3x + 4y - 3z = 2 \end{cases}$$

$$8) \begin{cases} x - y = 5 \\ y - z = -6 \\ 2x - z = 2 \end{cases} \quad \begin{cases} 2x - 4y + 3z = 10 \\ 3x + y - 2z = 6 \\ x + 3y - z = 20 \end{cases} \quad \begin{cases} 3x + y = 5 \\ 2y - 3z = -5 \\ x + 2z = 7 \end{cases}$$

$$9) \begin{cases} 2x - y - z - w = 0 \\ x - 3y + z + w = 0 \\ x + y - 4z + w = 0 \\ x + y + w = 36 \end{cases} \quad \begin{cases} x + y + 2z + w = 18 \\ x + 2y + z + w = 17 \\ x + y + z + 2w = 19 \\ 2x + y + z + w = 16 \end{cases}$$

$$10) \begin{cases} x + y = a + b \\ y + z = b + c \\ z + w = a - b \\ w - x = c - b \end{cases} \quad \begin{cases} x + 2y = 5 \\ y + 2z = 8 \\ z + 2u = 11 \\ 2x + u = 6 \end{cases} \quad \begin{cases} 3x - 2y - z + w = -3 \\ -x - y + 3z + 2w = 23 \\ x + 3y - 2z + w = -12 \\ 2x - y - z - 3w = -22 \end{cases}$$

$$11) \begin{cases} x + y + z + w = 4 \\ x - 2y + z - w = -1 \\ x + y + z + 2w = 5 \\ 2x - y + z - w = 1 \end{cases} \quad \begin{cases} 2x - 3y + z - w = -6 \\ x + 2y - z = 8 \\ 3y + z + 3w = 0 \\ 3x - y + w = 0 \end{cases} \quad \begin{cases} x + y = 4 \\ x - 2y + z = -1 \\ x + 2y + z = 5 \\ 2x - 4y + 2z = 1 \end{cases}$$

Chapter 4 . [Integrals]

Compute the following integrals:

$$1) \int (3x^2 + 1) \sin(x^3 + x + 1) dx ; \int \frac{x+3}{(x-3)(x-2)} dx ; \int \frac{5}{x^2 + 1} dx ;$$

$$2) \int (x^2 + 1) \ln x dx ; \int x^2 \sin x dx ; \int \frac{x}{(x+2)^2} dx ; \int x \sin x dx$$

$$3) \int \frac{dx}{x^2 + 9} ; \int x^2 \cos x dx ; \int \frac{x+1}{(x-1)^2} dx ; \int x^2 \ln x dx ; \int x \ln(x) dx ;$$

$$4) \int \frac{dx}{(x-3)(x-2)} ; \int (x+1) \cos(x^2 + 2x) dx ; \int \frac{x-2}{(x-2)(x-3)} dx ;$$

$$5) \int \frac{x-1}{(x+2)^2(x+1)} dx ; \int (2x+1) \sin x dx ; \int \frac{\sin x}{\cos x} dx ;$$

$$6) \int (3x^2 + 1) \sin(x^3 + x + 1) dx ; \int \frac{x+3}{(x-3)(x-2)} dx ; \int \frac{5}{x^2 + 1} dx ;$$

$$7) \int (x^2 + 1) \ln x dx ; \int \frac{2x+1}{(x+1)(x-4)} dx ; \int \frac{2x}{x^2 + 1} dx .$$

Chapter 5 . [Applications of Integrals]

Find the area of the surface determined by the curves:

$$1) x + y = 2, y = 2, y = 2x - 4 ;$$

$$2) x + y = 2, y = 2, y = 2x - 4 ;$$

$$3) y = 0, x = 1, x = 2, y = 4x^2 ;$$

$$4) x = 2, y = 0, y = -x^2 ;$$

$$5) y = 0, y = 2x + 2, y = -x + 1 ;$$

- 6) $x = 2, y = 0, y = x^2$;
- 7) $y = 0, y = -x + 6, y = \sqrt{x}$;
- 8) $x = 0, x = 1, y = 0, y = x^2 - 4$;
- 9) $x + y = 2, y = 2, y = 2x - 4$;
- 10) $x = \frac{1}{2}, x = 0, y = x, y = x^3$
- 11) $y = x, y = x^4$
- 12) $x = 0, x = \frac{\pi}{2}, y = 0, y = \cos x$
- 13) $y = 4, y = 0, x = 0, x = y^2 - 4y$
- 14) $x = y - 2, x^2 = y$
- 15) $x + y = 6, y = x^2 + 4$
- 16) $y = 4, y = -1, y = x - 6, y^2 = -x$
- 17) $y = -\frac{1}{5}x + 7, y = 2 + |x - 1|$
- 18) $x = 1, x = 0, y = \sqrt{x}, y = \frac{1}{2}x$
- 19) $y = \frac{1}{2}x, y = \sqrt{x}$
- 20) $x = 2 - y^2, x = y^2$

Find the volume of the solid of revolution generated by rotation about one of the coordinate axes of the region R limited by the following curves:

1. $x + y = 1, x = 1, y = 2x + 1$ (rotated about the y-axis);
2. $x = 0, x = 1, y = 1,$ and $y = x^2 + 2$ (rotated about the x-axis);

3. $x=0$, $y=-4x-4$ and $y = x^2$ (rotated about the y-axis);
4. $x = 0$, $x=1$, and $y = x^2 + 2$ (rotated about the x-axis);
5. $y = 0$, $x = 1$, $y = 5\sqrt{x}$ (rotated about the y-axis);
6. $y = 0$, $x = 3$, $x = 1$, $y = \frac{1}{x}$ (rotated about the x-axis);
7. $y = 2$, $y = x^2$ (rotated about the y-axis);
8. $y = 0$, $y = x^2 - 4x$ (rotated about the x-axis);
9. $2y = x$, $y^2 = x$ (rotated about the y-axis);
10. $y = 4 - x^2$, $y = x^2$ (rotated about the x-axis);
11. $y = 0$, $x = 1$, $y = x^3$ (rotated about the y-axis);
12. $x = 2$, $y = -2x + 3$, $y = 2x - 1$ (rotated about the y-axis);
13. $x = 0$, $y = 1$, $y^2 = x$ (rotated about the x-axis);
14. $y = 0$, $x = 1$, $y = x^2$ (rotated about the x-axis);
15. $y = 0$, $x = 9$, $x = 4$, $y = \sqrt{x}$ (rotated about the x-axis);
16. $y = 0$, $y - x + 2 = 0$, $x = y^2$ (rotated about the y-axis);
17. $y - x + 2 = 0$, $x = y^2$ (rotated about the x-axis);