GENERAL MATHEMATICS 2

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Department of Mathematics

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Main Contents

Functions of Several Variables

(1) Functions of one variable.

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$$x \longrightarrow \omega \ .$$

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Functions of Several Variables

Definition

1. *A function of two variables is a rule that assigns an ordered pair* (x_1, x_2) *to a real number* w*:*

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$$

$$(x_1, x_2) \longrightarrow \omega \ .$$

2. *A function of three variables is a rule that assigns an ordered triple* (x_1, x_2, x_3) *to a real number* w*:*

$$f : \mathbb{R}^3 \longrightarrow \mathbb{R}$$

$$(x_1, x_2, x_3) \longrightarrow \omega \ .$$

3. *A function of* n *variables is a rule that assigns an ordered* n*-tuple* $(x_1, x_2, \dots, x_n)$ *to a real number* w*:*

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Example : $f(x, y, z, u, v) = x^2 + y^2 - 7zu + v^2$ is a function of five variables.

It takes $(x, y, z, u, v) \in \mathbb{R}^5$ to $\omega \in \mathbb{R}$, for example, the function f takes $(1, 0, 1, 1, 2) \in \mathbb{R}^5$ to $-2 \in \mathbb{R}$.

First Partial Derivative

Definition

Let $w = f(x, y)$ be a function of two variables.

1. *The partial derivative of $w = f(x, y)$ with respect to x denoted $\frac{\partial f}{\partial x}$, $\frac{\partial w}{\partial x}$, f_x or w_x is calculated by applying the rules of differentiation to x holding y constant.*
2. *The partial derivative of $w = f(x, y)$ with respect to y denoted $\frac{\partial f}{\partial y}$, $\frac{\partial w}{\partial y}$, f_y or w_y is calculated by applying the rules of differentiation to y holding x constant.*

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2. $f(x, y, z) = 2xy^{-1} + \sin(xy) + 4z^3 \Rightarrow f_y = 2x(-1)y^{-2} + \cos(xy)\ x + 0 = -\frac{2x}{y^2} + x\ \cos(xy)$.
3. $f_z = 0 + 0 + 4\ (3z^2)$

First Partial Derivative

Example

If $f(x, y, z) = \frac{2x}{y} + \sin(xy) + 4z^3$, *calculate* **(1)** f_x **(2)** f_y **(3)** f_z.

Solution:

1. $f_x = \frac{2}{y} + \cos(xy)\ y + 0 = \frac{2}{y} + y\ \cos(xy)$.
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Second Partial Derivative

- $\frac{\partial^2 f}{\partial x^2}$ means the second derivative with respect to x holding y constant.
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Let $f(x, y)$ be a function of two variables, then

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1. $f_x = 3x^2 + 4xy^2$, then $f_{xy} = 4x\,(2y) = 8xy$.

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Example

If $f(x,y,z) = z^3x + y^2(x + yz)$, *calculate*

1. f_x, f_y *and* f_z *at* $(1,1,1)$.
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