29.5 Gauss's Law in Magnetism

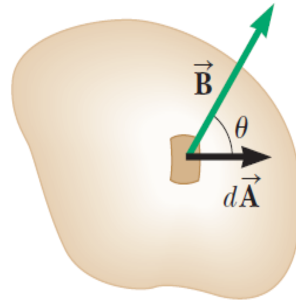


Figure 29.19 The magnetic flux through an area element dA is $\vec{B} \cdot d\vec{A} = B\,dA\cos\theta$, where $d\vec{A}$ is a vector perpendicular to the surface.

Magnetic flux:

The flux associated with the magnetic field is defined in a manner similar to that used to define electric flux.

In the figure, one can define the magnetic flux through the element area A as,

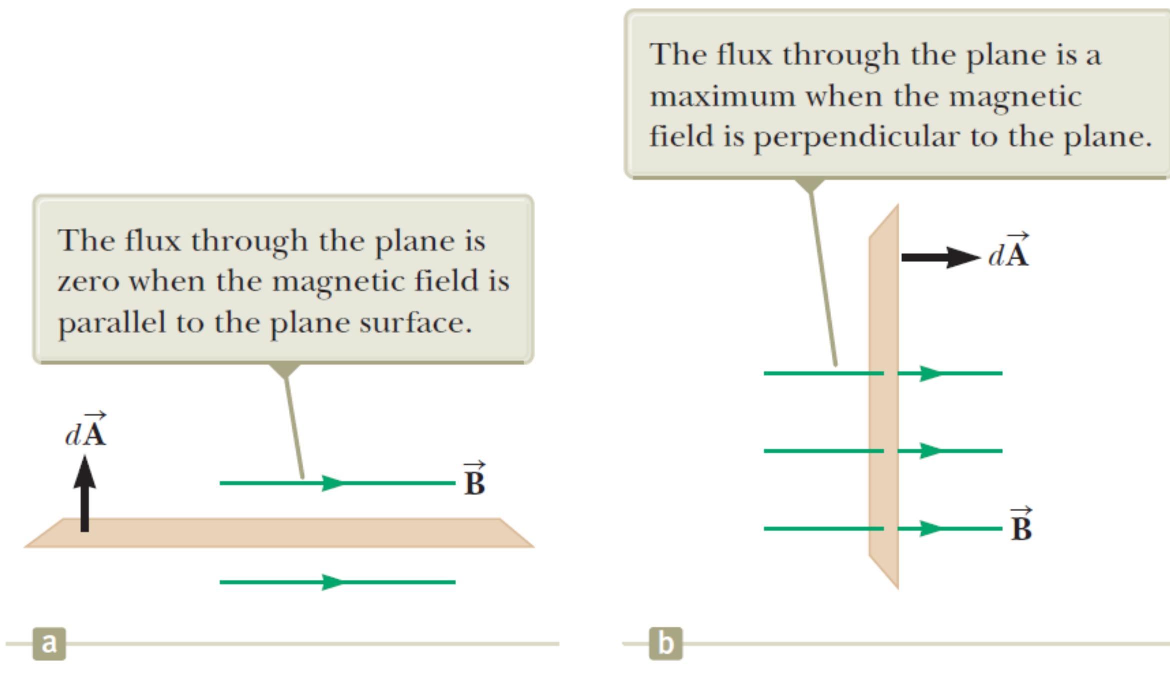
$$\Phi_B = \int \vec{B}.d\vec{A} \qquad 29.11$$

Special case, for a plane of area A in a uniform field B,

$$\Phi_B = BA\cos\theta$$

! Remember that θ is the angle between the field B and the vector $d\vec{A}$ that is perpendicular to the surface.

The unit of flux is $T.m^2$, which defined as a Weber (Wb); 1 $Wb=T.m^2$



$\Phi_B = 0$

$\Phi_B = BA$

In Chapter 23 we found that the electric flux through a closed surface surrounding a net charge is proportional to that charge (Gauss's law). In other words, the number of electric field lines leaving the surface depends only on the net charge within it. This property is based on the fact that electric field lines originate and terminate on electric charges. The situation is quite different for magnetic fields, which are continuous and form closed loops. The number of lines entering the surface equals the number leaving the surface; therefore, the net magnetic flux is zero.

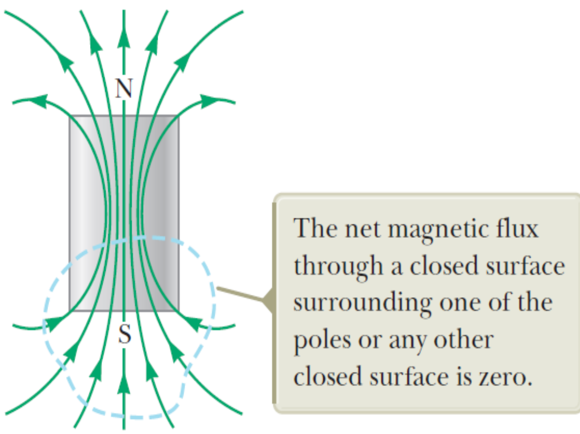


Figure 29.22 The magnetic field lines of a bar magnet form closed loops. (The dashed line represents the intersection of a closed surface with the page.)

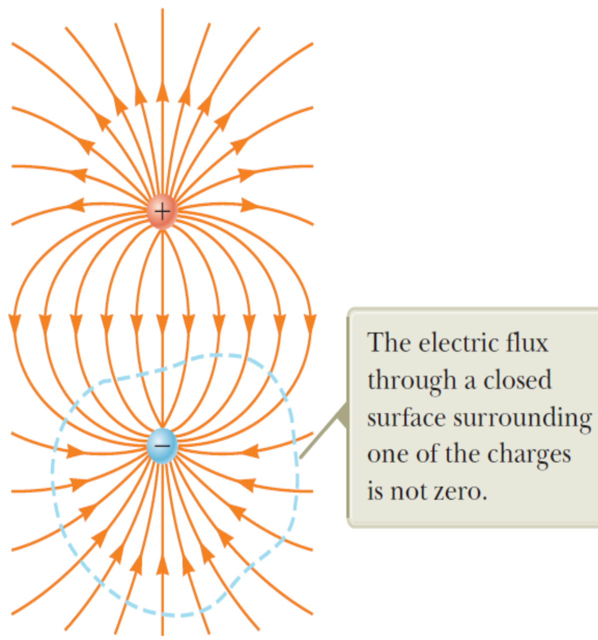


Figure 29.23 The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge.

Gauss's law in magnetism states that "the net magnetic flux through any closed surface is always zero:

$$\oint \vec{B}.d\vec{A} = 0 \qquad 29.12$$

Remember: **For the Electric field**:

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\epsilon_0}$$

Examples:

Q1- If the magnetic flux through a closed surface is always zero, this means:

A) There are no electric currents inside the surface.
B) The net charge enclosed is zero.
C) The number of magnetic field lines entering the surface equals the number leaving it.
D) The magnetic field inside is necessarily zero.

Q2- What does Gauss's Law for magnetism imply about the nature of magnetic field lines?

A) Magnetic field lines always form closed loops.
B) Magnetic field lines originate from positive charges and end at negative charges.
C) Magnetic field lines can begin or end inside a material.
D) Magnetic field lines can exist only in vacuum.

Q3- If a conducting loop is placed in a uniform magnetic field with no change in field strength or orientation, what happens to the magnetic flux?

A) It increases.
B) It decreases.
C) It remains constant.
D) It oscillates.

Q4- A rectangular coil with dimensions 0.30 m × 0.20 m lies in a uniform magnetic field of magnitude 0.40 T. The magnetic field makes an angle of 60° with the normal to the plane of the coil.

1. Find the magnetic flux through the coil.

Solution:

Given:
$L = 0.30$ m, $W = 0.20$ m, $B = 0.40$ T, $\theta = 60°$, $N = 50$

1. **Magnetic Flux:**
$A = L \times W = 0.30 \times 0.20 = 0.060\ m^2$
$\Phi = B \times A \times \cos(\theta) = 0.40 \times 0.060 \times \cos(60°) = 0.40 \times 0.060 \times 0.5 = 1.2\times10^{-2}$ Wb