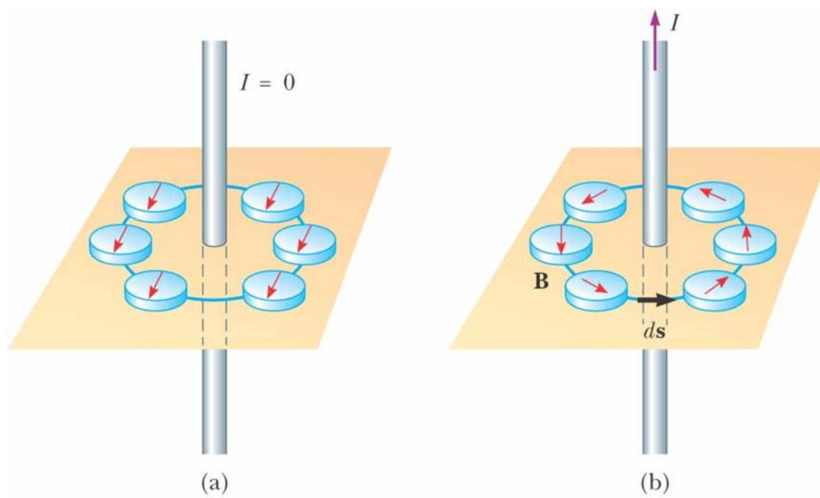
## 30.3 Ampere's Law

Because the compass needles point in the direction of B, we conclude that the lines of B form circles around the wire,

- The magnitude of B is the same everywhere on a circular path centered on the wire and lying in a plane perpendicular to the wire.
- By varying the current and distance *a from the wire, we find that B is proportional to the current and inversely* proportional to the distance from the wire,





$$\oint \vec{B}.d\vec{s} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I \qquad 30.7$$

Ampere's law states that "the line integral of $\vec{B}.d\vec{s}$ around any closed path equals $\mu_0 I$ , where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{B}.d\vec{s} = \mu_0 I$$

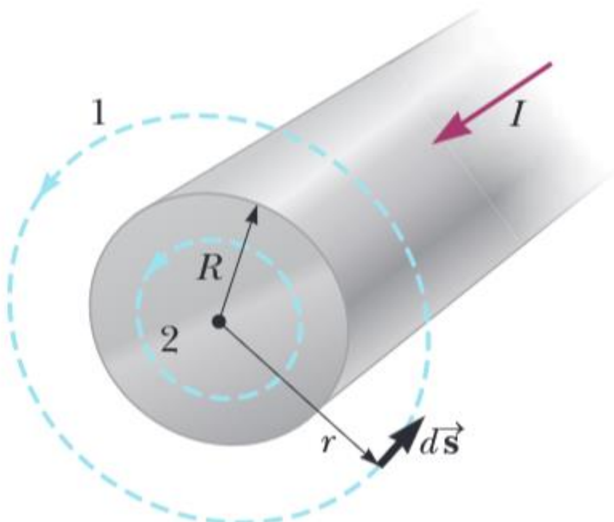
Ampère's Law is a fundamental equation in electromagnetism that relates the **magnetic field (B)** around a closed loop to the total current passing through the loop. It is analogous to **Gauss's Law** in electrostatics but applies to magnetic fields.

**Example**:

**The Magnetic Field Created by a Long Current-Carrying Wire:**

**A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (see the Fig. ). Calculate the magnetic field a distance r from the center of the wire in the regions r ≥ R and r < R.**

Let us choose for our path of integration circle 1 in the Figure in front from symmetry, B must be constant in magnitude and parallel to *ds at every* point on this circle. Because the total current passing through the plane of the circle is $I_0$, *Ampère's law gives* :




$$\oint \vec{B}.d\vec{s} = B\oint ds = B(2\pi r) = \mu_0 I_0$$

$$=== > \boldsymbol{B = \frac{\mu_0 I_0}{2\pi r} \alpha \frac{1}{r}}$$ This is for r ≥ R

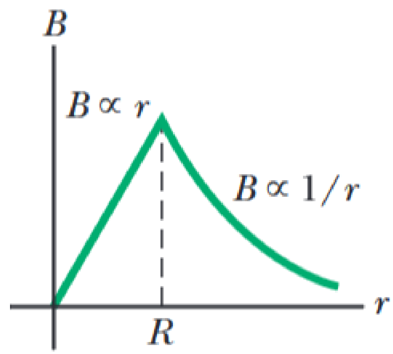
Now consider the interior of the wire, where r <R. Here the current I passing through the plane of circle 2 is less than the total current $I_0$. Because the current is uniform over the cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area $r^2$ enclosed by circle 2 to the cross-sectional area $R^2$ of the wire:

$$\frac{I}{I_0} = \frac{\pi r^2}{\pi R^2} \quad === > \quad I = \frac{r^2}{R^2} I_0$$

$$\oint \vec{B}.d\vec{s} = B\oint ds = B(2\pi r) = \mu_0 I = \mu_0 (\frac{r^2}{R^2} I_0)$$

$$=== > \quad B = (\frac{\mu_0 I_0}{2\pi R^2}) r$$ This is for r < R

**Figure 29.14** (Example 29.5) Magnitude of the magnetic field versus $r$ for the wire shown in Figure 29.13. The field is proportional to $r$ inside the wire and varies as $1/r$ outside the wire.




**Example 1: A long straight wire carries a 10 A current. Find the magnetic field at a point 5 cm away from the wire.**

**Example 2: Ampère's Law states that the line integral of the magnetic field around a closed path is proportional to:**

A) The electric flux passing through the surface
B) The total current enclosed by the loop
C) The total charge enclosed by the loop
D) The resistance of the wire

**Example 3: If the current in a long straight wire is doubled, how does the magnetic field at a fixed distance change?**

A) It remains the same
B) It doubles
C) It halves
D) It becomes four times larger

**Example 4: A long cylindrical conductor of radius R carries a steady current I uniformly distributed over its cross-section. Using Ampère's Law, determine the ratio of the magnetic field at $r = \frac{R}{2}$ to the magnetic field at r=R.**

A) 1/2
B) 1/4
C) 2/3
D) 3/4