29.3 Ampere's Law

Because the compass needles point in the direction of **B**, we conclude that the lines of B form circles around the wire.

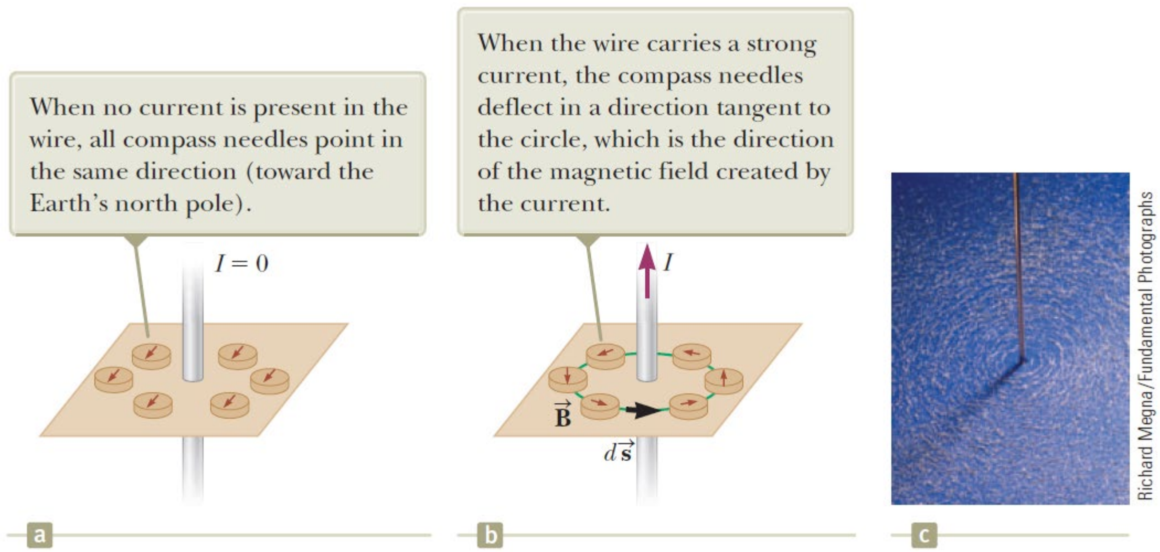


Figure 29.10 (a) and (b) Compasses show the effects of the current in a nearby wire. (c) Circular magnetic field lines surrounding a current-carrying conductor, displayed with iron filings.

- The magnitude of **B** is the same everywhere on a circular path centered on the wire and lying in a plane perpendicular to the wire.
- By varying the current and distance *a from the wire, we find that B is proportional to the current and inversely* proportional to the distance from the wire,

Note that the vectors **ds** and **B** are parallel at each point.

- Symmetry → B constant
- B parallel to path

$$\oint \vec{B}.d\vec{s} = \vec{B}\oint d\vec{s} = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I_{enclsoed} \qquad 29.7$$

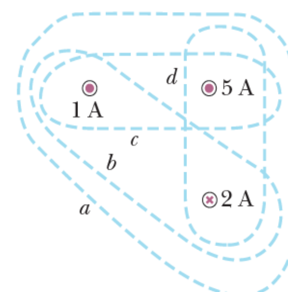
Ampere's law states that "the line integral of $\vec{B}.d\vec{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{B}.d\vec{s} = \mu_0 I_{enclsoed}$$

Concept: Only the net enclosed current affects the integral, regardless of the path's shape or size.

Ampère's Law is a fundamental equation in electromagnetism that relates the **magnetic field (B)** around a closed loop to the total current passing through the loop. It is analogous to **Gauss's Law** in electrostatics but applies to magnetic fields.

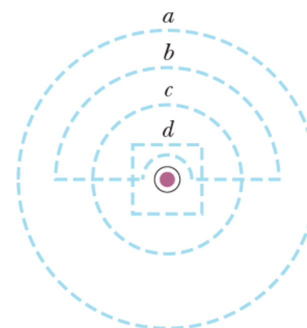
Example-1: Rank the magnitudes of **B.ds** for the closed paths a through d in the Figure from the greatest to least.



Solution:
For loop (a): $|5 + 1 - 2| = 2\ A$
For loop (b): $|1 - 2| = 1\ A$
For loop (c): $|5 + 1| = 6\ A$
For loop (d): $|5 - 2| = 3\ A$

Then: c,a,d,b

Example-2: Rank the magnitudes of **B.ds** for the closed paths a through d in the Figure from greatest to least.

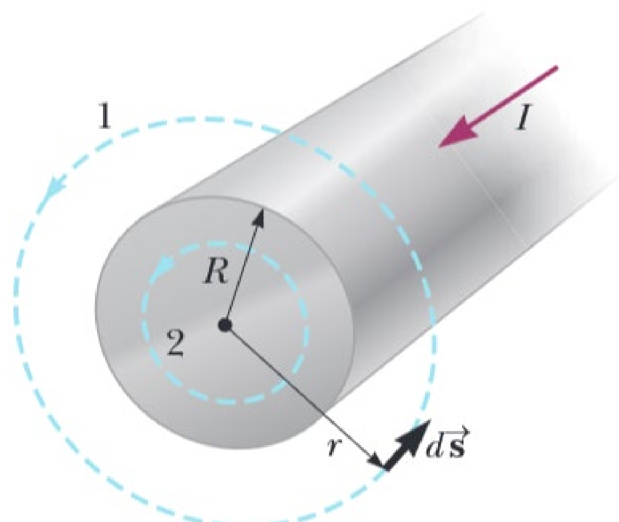


The Magnetic Field Created by a Long Current-Carrying Wire:

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (see the figure).

Calculate the magnetic field at a distance r from the center of the wire in the regions r ≥ R and
r < R.

Let us choose the path of integration circle 1 in the Figure in front, based on symmetry. B must be constant in magnitude and parallel to *ds at every* point on this circle. Because the total current passing through the plane of the circle is I_0, *Ampère's law gives*:



$$\oint \vec{B}.d\vec{s} = B\oint ds = B(2\pi r) = \mu_0 I_0$$

$$===> \boldsymbol{B} = \frac{\boldsymbol{\mu_0 I_0}}{\boldsymbol{2\pi r}}\ \boldsymbol{\alpha}\ \frac{\mathbf{1}}{\boldsymbol{r}}$$ This is for r ≥ R

Now consider the interior of the wire, where r <R. Here, the current I passing through the plane of circle 2 is less than the total current I_0. Because the current is uniform over the cross-section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area r^2 enclosed by circle 2 to the cross-sectional area R^2 of the wire:

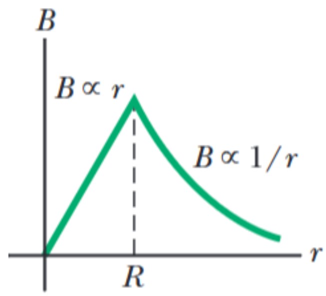
Current per unit area is constant (uniform distribution). That is the density of current $(J = \frac{I}{A})$

$$\frac{I}{I_0} = \frac{\pi r^2}{\pi R^2} \quad ===> \quad I = \frac{r^2}{R^2} I_0$$

$$\oint \vec{B}.d\vec{s} = B \oint ds = B(2\pi r) = \mu_0 I = \mu_0 (\frac{r^2}{R^2} I_0)$$

$===> \quad B = (\frac{\mu_0 I_0}{2\pi R^2}) r$ This is for r < R

Figure 29.14 (Example 29.5) Magnitude of the magnetic field versus *r* for the wire shown in Figure 29.13. The field is proportional to *r* inside the wire and varies as 1/*r* outside the wire.



- **The field is proportional to r inside the wire.**
- **The field varies as 1/r outside the wire.**
- **Both equations are equal at r = R.**

Example 3: A long straight wire carries a 10 A current. Find the magnetic field at a point 5 cm away from the wire.

Example 4: Ampère's Law states that the line integral of the magnetic field around a closed path is proportional to:

A) The electric flux passing through the surface
B) The total current enclosed by the loop
C) The total charge enclosed by the loop
D) The resistance of the wire

Example 5: If the current in a long straight wire is doubled, how does the magnetic field at a fixed distance change?

A) It remains the same
B) It doubles
C) It halves
D) It becomes four times larger

Example 6: A long cylindrical conductor of radius R carries a steady current I uniformly distributed over its cross-section. Using Ampère's Law, determine the ratio of the magnetic field at $r = \frac{R}{2}$ to the magnetic field at r=R.

A) 1/2
B) 1/4
C) 2/3
D) 3/4