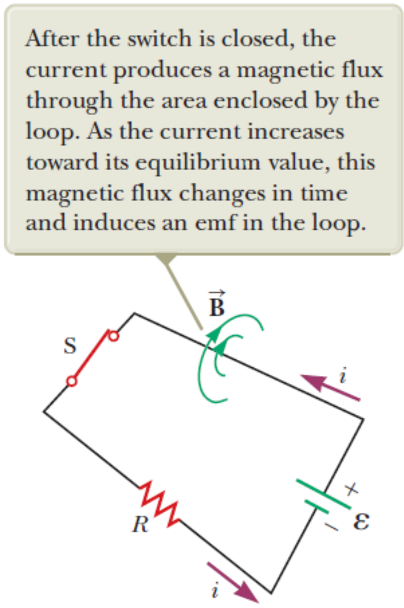
Chapter 32

Inductance

32.1 Self-Induction and Inductance:

- **Definition**: Self-induction is the phenomenon where a changing current in a circuit induces an electromotive force (emf) in the same circuit.

- When the current through a coil changes, the magnetic field created by the coil also changes, inducing a **back emf** that opposes the change in current (Lenz's Law).



Simulations:

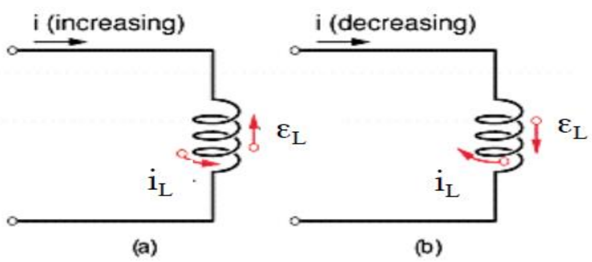
https://www.youtube.com/watch?v=1F1ssAiPYC8

As the source current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. This increasing flux creates an induced emf in the circuit.

The direction of the induced emf is such that it would cause an induced current in the loop (if a current were not already flowing in the loop), which would establish a magnetic field that would oppose the change in the source magnetic field.

After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop.

As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop.



$$\Phi_B \;\alpha\; I$$
$$\Phi_B = K\, I$$

$$\varepsilon_L = -N\frac{d\Phi}{dt} = -N\frac{d\Phi}{dI}\frac{dI}{dt}$$
$$= -N\,K\frac{dI}{dt}$$

dI

Using Faraday's law, the self-induced emf can be written as,

32.1

Where L is the proportionality constant-called the inductance of the loop that depends on the geometry of the loop and other physical characteristics.

- The minus sign reflects **Lenz's Law**—the induced emf opposes the change in current.

We know that: $\varepsilon_L = -N\frac{d\Phi_B}{dt}$ so we substitute this in eq. (32.1) as following,

$$-N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$
$$\Rightarrow L = \frac{N\Phi_B}{I} \qquad 32.2$$

The unit of L is henry (H).

Example-1:

Consider a uniformly wound solenoid having *N* turns and length *l*. Assume *l* is much longer than the radius of the windings and the core of the solenoid is air.

(A) Find the inductance of the solenoid.

(B) Calculate the inductance of the solenoid if it contains 300 turns, its length is 25.0 cm, and its cross-sectional area is 4.00 cm^2.

(C) Calculate the self-induced emf in the solenoid if the current it carries decreases at the rate of 50.0 A/s.

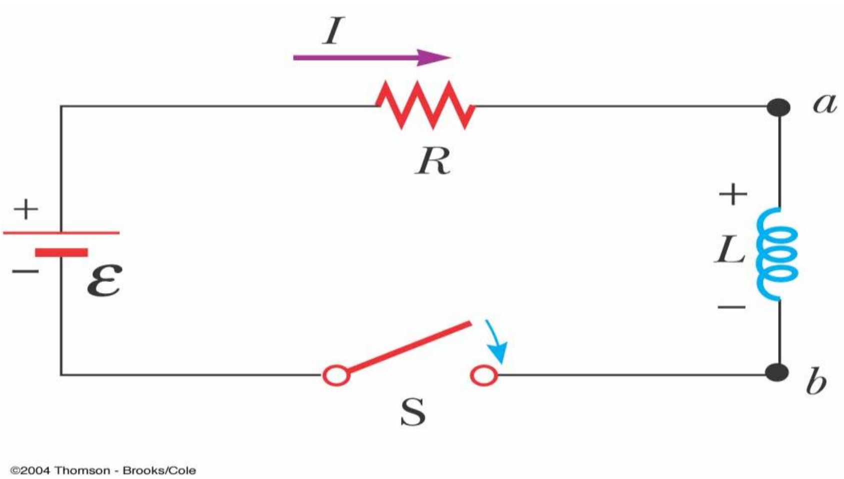
Exampl-2:

An emf of 24.0 mV is induced in a 500− turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0/through each turn of the coil?

$$\varepsilon = L\frac{dI}{dt} ==> L = \frac{\varepsilon}{\frac{\Delta I}{\Delta t}} = \frac{24x10^{-3}}{10} = 2.40mH$$

Since, $L = N\frac{\Phi}{I}$ $==> \Phi = \frac{IL}{N} = \frac{4x2.4x10^{-3}}{500} = 1.92x10^{-5}\ T.m^2$

32.2 Energy in Magnetic field:



In the figure, the battery in the circuit must provide more energy than in a circuit without the inductor. The energy supplied by the battery appears as internal energy in the resistance and the remaining energy is stored in the magnetic field of the inductor. This can be formulated as following,

$$\varepsilon = IR + L\frac{dI}{dt} \qquad 32.3$$

Multiplying this equation by I,

$$I\varepsilon = I^2R + LI\frac{dI}{dt} \qquad 32.4$$

Ok, what does this equation represent?

The last term $\left(LI\frac{dI}{dt}\right)$ represents the rate at which energy U is being stored in the indictor.

$$\frac{dU}{dt} = LI\frac{dI}{dt} \qquad 32.5$$

By integrating this equation one can find the total energy stored in the inductor at any instant,

$$U = \int dU = L\int_0^I IdI = \frac{1}{2}LI^2 \qquad 32.5$$

Also, we can find the energy density of a magnetic field created by a solenoid. For a solenoid, we have derived the magnetic field and the inductance:

$$L = \mu_0 n^2 V$$

$$B = \mu_0 n\ I \Rightarrow I = \frac{B}{\mu_0 n}$$

Substituting these in eq. (32.5),

$$U = \frac{1}{2}\left(\mu_0 n^2 V\right)\left(\frac{B}{\mu_0 n}\right)^2 = \frac{1}{2\mu_0}B^2 V$$

$$u_B = \frac{U}{V} = \frac{1}{2\mu_0}B^2 \qquad 32.6$$

Where u_B indicates the energy density.

Example-3:

Calculate the energy associated with the magnetic field of a 200 − turn solenoid in which a current of 1.75 A produces a flux of 3.70 × 10−4 Wb in each turn.

$U = \frac{1}{2}LI^2$ L = ?

$$L = \frac{N}{I} = \frac{200(3.7x10^{-4})}{1.75} = 42.3\ mH$$

$==> U = \frac{1}{2}LI^2 = \frac{1}{2}(42.3x10^{-3})(1.75)^2 = 0.0648\,J$

Applications:

🔌 Applications of Self-Inductance

Self-inductance plays a key role in many electrical and electronic systems. Below are some practical and engineering applications:

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1. Inductors in Electronic Circuits

- **Purpose**: Store energy, filter signals, or limit current.
- **How**: Inductors resist sudden changes in current, making them useful in timing circuits, signal processing, and voltage regulation.

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2. Transformers (Indirect application)

- **Although mainly mutual inductance**, self-inductance is important in each coil.
- Transformers rely on the inductance of their coils to regulate how voltage and current are transferred between primary and secondary circuits.

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3. Ignition Systems in Vehicles

- **Purpose**: Generate high voltage to ignite fuel.
- **How**: A coil stores energy through self-inductance. When the current is suddenly interrupted, the collapsing magnetic field induces a large voltage spike, firing the spark plug.

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4. Switching Power Supplies (SMPS)

- Inductors help maintain steady current flow when transistors switch on/off rapidly.
- Self-inductance smooths out current spikes.

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5. Relay and Motor Protection

- When relays or motors are turned off, the stored magnetic energy (due to self-inductance) can cause damaging voltage spikes.
- **Solution**: Diodes (called flyback or freewheeling diodes) are used to safely dissipate this energy.

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6. Wireless Charging Systems

- The charging coil has inductance, and the changing current in one coil induces emf in the receiver coil via mutual and self-inductance principles.

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7. Inductive Sensors

- Used in metal detectors, speed sensors (like in ABS braking systems), and traffic light sensors.
- Relies on changes in inductance due to nearby metal objects.

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8. Audio Equipment

- Inductors in crossover networks of speakers filter certain frequency ranges (like bass, treble) using self-inductive properties.
