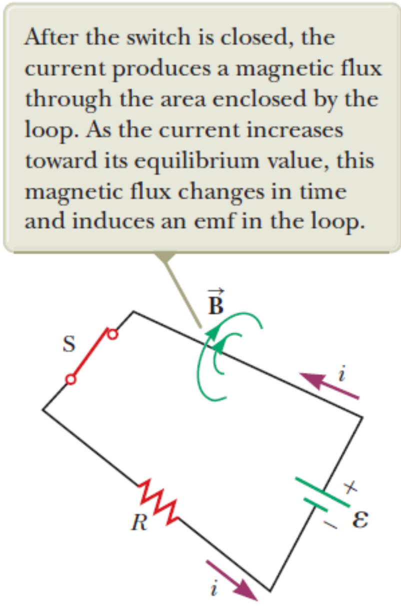
# Chapter 32

# Inductance

## 32.1 Self-Induction and Inductance:

- **Definition**: Self-induction is the phenomenon where a changing current in a circuit induces an electromotive force (emf) in the same circuit.

- When the current through a coil changes, the magnetic field created by the coil also changes, inducing a **back emf** that opposes the change in current (Lenz's Law).




## Simulations:

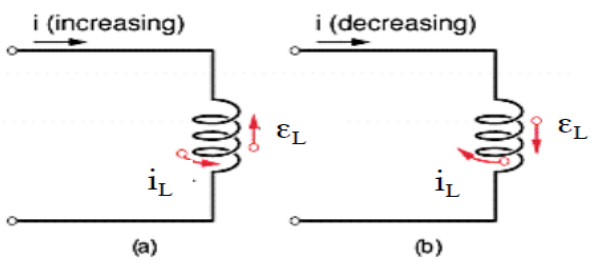
**https://www.youtube.com/watch?v=1F1ssAiPYC8**

As the source current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. This increasing flux creates an induced emf in the circuit.

The direction of the induced emf is such that it would cause an induced current in the loop (if a current were not already flowing in the loop), which would establish a magnetic field that would oppose the change in the source magnetic field.

After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop.

As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop.



Using Faraday's law, the self-induced emf can be written as,

$$\Phi_B \ \alpha I$$
$$\Phi_B = K\ I \qquad 32.1$$

$$\varepsilon_L = -N\frac{d\Phi}{dt} = -N\frac{d\Phi}{dI}\frac{dI}{dt}$$
$$= -N\ K\frac{dI}{dt}$$
$$\varepsilon_L = -L\ \frac{dI}{dt}$$

Where **L** is the proportionality constant, called the inductance of the loop, that depends on the geometry of the loop and other physical characteristics.

- The minus sign reflects **Lenz's Law**—the induced emf opposes the change in current.

We know that: $\varepsilon_L = -N\frac{d\Phi_B}{dt}$ so we substitute this in eq. (32.1) as follows,

$$-N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt} \qquad 32.2$$
$$\Rightarrow L = \frac{N\Phi_B}{I}$$

The unit of L is henry (H).

**Some Terminology:**

- Use *emf* and *current* when batteries or other sources cause them.
- Use *induced emf* and *induced current* when they are caused by changing magnetic fields.

When dealing with problems in electromagnetism, it is essential to distinguish between the two situations.

## Think about it:

- When the current in an inductor decreases with time, the induced emf:

A. Acts in a direction that opposes the decrease in current, making it positive. ☑
B. Acts in the same direction as the change in current, making it negative.
C. Becomes zero because the current is decreasing.
D. Always reverses the direction of current in the circuit.

**Example-1:**

Consider a uniformly wound solenoid having *N* turns and length *l*. Assume *l* is much longer than the radius of the windings and the core of the solenoid is air.

**(A)** Find the inductance of the solenoid.

The magnetic flux through each turn of cross-sectional area A is given by:

$\Phi_B = B \cdot A = \mu_o \cdot n \cdot I \cdot A = \mu_o (N/\ell)\, I\, A$

The inductance is:

$L = N\Phi_B / I = \mu_o N^2 A / \ell = \mu_o n^2 V$

**(B)** Calculate the inductance of the solenoid if it contains 300 turns, its length is 25.0 cm, and its cross-sectional area is 4.00 $cm^2$.

**Solution:**

$L = \mu_o N^2 A / \ell$

$= (4\pi \times 10^{-7}\ T{\cdot}m/A)(300)^2(4.00 \times 10^{-4}\ m^2) / (25.0 \times 10^{-2}\ m)$

$= 1.81 \times 10^{-4}\ T{\cdot}m^2/A = 0.181\ mH$

**(C)** Calculate the self-induced emf in the solenoid if the current it carries decreases at the rate of 50.0 A/s.

**Solution:**

$\varepsilon_L = -L\,(dI/dt)$

$= -(1.81 \times 10^{-4}\ H)(-50.0\ A/s)$

$= 9.05\ mV$

**Exampl-2:**

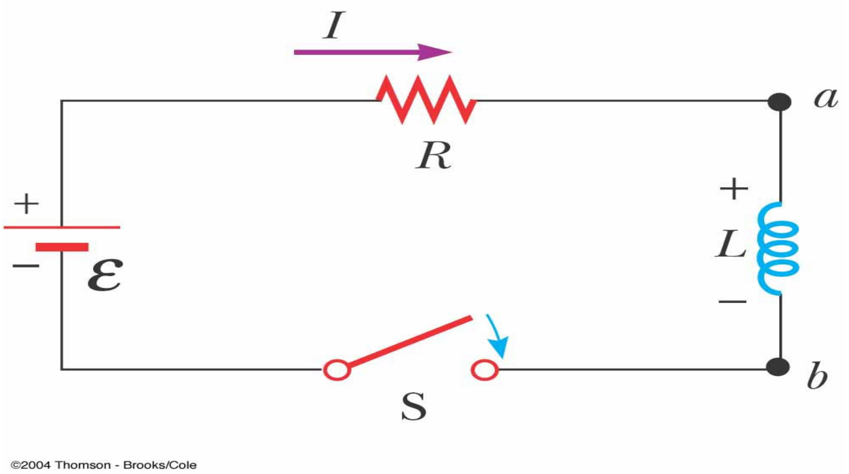
An emf of 24.0 mV is induced in a 500− turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?

$$\varepsilon = L\frac{dI}{dt} ==> L = \frac{\varepsilon}{\frac{\Delta I}{\Delta t}} = \frac{24x10^{-3}}{10} = 2.40mH$$

Since, $L = N\frac{\Phi}{I}$ $\quad ==> \Phi = \frac{IL}{N} = \frac{4x2.4x10^{-3}}{500} = 1.92x10^{-5}\ T.m^2$

## 32.2 Energy in a Magnetic field:

If a circuit contains a coil such as a solenoid, the inductance of the coil prevents the current in the circuit from increasing or decreasing instantaneously. A circuit element that has a large inductance is called an **inductor** and has the circuit symbol (). We always assume the inductance of the remainder of a circuit is negligible compared with that of the inductor.




In the figure, the battery must provide more energy than in a circuit without the inductor. The energy supplied by the battery appears as internal energy in the resistor, and the remaining energy is stored in the inductor's magnetic field. This can be formulated as follows,

$$\varepsilon = IR + L\frac{dI}{dt} \qquad 32.3$$

Multiplying this equation by I,

$$I\varepsilon = I^2R + LI\frac{dI}{dt} \qquad 32.4$$

Ok, what does this equation represent?

The last term $\left(LI\frac{dI}{dt}\right)$ represents the rate at which energy U is being stored in the indictor (recall the definition of the power).

$$\frac{dU}{dt} = LI\frac{dI}{dt} \qquad 32.5$$

By integrating this equation, one can find the total energy stored in the inductor at any instant,

$$U = \int dU = L\int_0^I IdI = \frac{1}{2}LI^2 \qquad 32.5$$

Also, we can find the energy density of a magnetic field created by a solenoid. **For a solenoid**, we have derived the magnetic field and the inductance:

$$L = \mu_0 n^2 V$$

$$B = \mu_0 n\ I \Rightarrow I = \frac{B}{\mu_0 n}$$

Substituting these in eq. (32.5),

$$U = \frac{1}{2}(\mu_0 n^2 V)\left(\frac{B}{\mu_0 n}\right)^2 = \frac{1}{2\mu_0}B^2 V$$

$$u_B = \frac{U}{V} = \frac{1}{2\mu_0}B^2 \qquad 32.6$$

Where $u_B$ indicates the energy density.

**Example-3:**

**Calculate the energy associated with the magnetic field of a 200 − turn solenoid in which a current of 1.75 A produces a flux of 3.70 × $10^{-4}$ Wb in each turn.**

$U = \frac{1}{2}LI^2$ L = ?

$$L = \frac{N\Phi}{I} = \frac{200(3.7x10^{-4})}{1.75} = 42.3\,mH$$

$$==> U = \frac{1}{2}LI^2 = \frac{1}{2}(42.3x10^{-3})(1.75)^2 = 0.0648\,J$$

## Problem 32.29

**Calculate the energy associated with the magnetic field of a 200-turn solenoid in which a current of 1.75 A produces a flux of $3.70 \times 10^{-4}$ Wb in each turn.**

## Solution

Inductance from flux linkage:

$L = (N\,\Phi_B) / I = (200 \times 3.70 \times 10^{-4}) / 1.75 = 4.23 \times 10^{-2}$ H = 42.3 mH.

Energy stored in the magnetic field:

$U = (1/2)\,L\,I^2 = (1/2)(0.0423\text{ H})(1.75\text{ A})^2 = 0.0648$ J.

## Problem 32.30

**The magnetic field inside a superconducting solenoid is 4.50 T. The solenoid has an inner diameter of 6.20 cm and a length of 26.0 cm. Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.**

## Solution

(a) Magnetic energy density: $u = B^2 / (2\,\mu_0)$.

$u = (4.50\text{ T})^2 / [2\,(1.26 \times 10^{-6}\text{ T·m/A})] = 8.06 \times 10^6\text{ J/m}^3$.

(b) Energy stored: $U = u\,V$, where V is the volume occupied by the field.

Inner radius $r = (6.20\text{ cm})/2 = 3.10\text{ cm} = 0.0310$ m; length $\ell = 0.260$ m.

$V = \ell\,\pi\,r^2 = (0.260\text{ m})\,\pi\,(0.0310\text{ m})^2 = 7.85 \times 10^{-4}\text{ m}^3$.

$U = (8.06 \times 10^6\text{ J/m}^3)(7.85 \times 10^{-4}\text{ m}^3) = 6.32 \times 10^3\text{ J} = 6.32$ kJ.

## Problem 32.31

**An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. How much energy is stored in its magnetic field when it carries a current of 0.770 A?**

## Solution

Inductance of a long solenoid: $L = \mu_0 N^2 A / \ell$, with $A = \pi r^2$ and $r = 0.600$ cm $= 0.00600$ m.

$A = \pi (0.00600 \text{ m})^2 = 1.131 \times 10^{-4} \text{ m}^2$.

$L = (4\pi \times 10^{-7} \text{ H/m}) (68)^2 (1.131 \times 10^{-4} \text{ m}^2) / 0.0800 \text{ m} = 8.21 \times 10^{-6} \text{ H} = 8.21\ \mu\text{H}$.

Energy stored:

$U = (1/2) L I^2 = (1/2)(8.21 \times 10^{-6} \text{ H})(0.770 \text{ A})^2 = 2.44 \times 10^{-6} \text{ J} = 2.44\ \mu\text{J}$.

# Applications:

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## 🔌 Applications of Self-Inductance

Self-inductance plays a key role in many electrical and electronic systems. Below are some practical and engineering applications:

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### 1. Inductors in Electronic Circuits

- **Purpose**: Store energy, filter signals, or limit current.
- **How**: Inductors resist sudden changes in current, making them useful in timing circuits, signal processing, and voltage regulation.

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### 2. Transformers (Indirect application)

- **Although mainly mutual inductance**, self-inductance is important in each coil.
- Transformers rely on the inductance of their coils to regulate how voltage and current are transferred between primary and secondary circuits.

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### 3. Ignition Systems in Vehicles

- **Purpose**: Generate high voltage to ignite fuel.
- **How**: A coil stores energy through self-inductance. When the current is suddenly interrupted, the collapsing magnetic field induces a large voltage spike, firing the spark plug.

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### 4. Switching Power Supplies (SMPS)

- Inductors help maintain steady current flow when transistors switch on/off rapidly.
- Self-inductance smooths out current spikes.

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### 5. Relay and Motor Protection

- When relays or motors are turned off, the stored magnetic energy (due to self-inductance) can cause damaging voltage spikes.
- **Solution**: Diodes (called flyback or freewheeling diodes) are used to safely dissipate this energy.

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### 6. Wireless Charging Systems

- The charging coil has inductance, and the changing current in one coil induces emf in the receiver coil via mutual and self-inductance principles.

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### 7. Inductive Sensors

- Used in metal detectors, speed sensors (like in ABS braking systems), and traffic light sensors.
- Relies on changes in inductance due to nearby metal objects.

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### 8. Audio Equipment

- Inductors in crossover networks of speakers filter certain frequency ranges (like bass, treble) using self-inductive properties.

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