Chapter 29

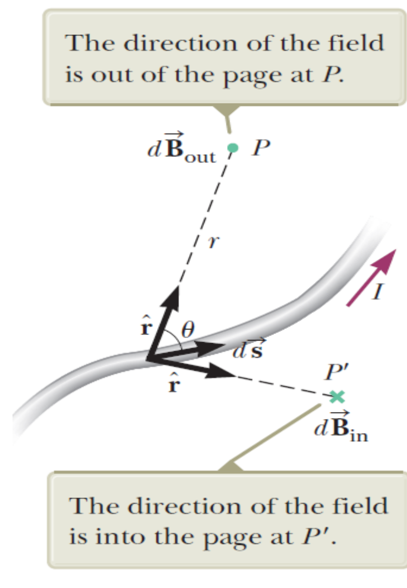
Sources of the Magnetic Field

We discussed the magnetic force exerted on a charged particle moving in a magnetic field.

In this chapter, the origin of the magnetic field—moving charges —will be discussed. We begin by showing how to use the Biot-Savart law to calculate the magnetic field at a point in space due to a small current element.

29.1 The Biot-Savart Law

- The Biot-Savart Law provides a way to calculate the **magnetic field** at any point due to a small current element.
- It helps derive expressions for magnetic fields in various geometries, such as **straight wires, circular loops, and solenoids**.



Jean-Baptiste Biot and Félix Savart performed quantitative experiments on the force exerted by an electric current on a nearby magnet. The experimental observations for the magnetic field **dB** at a point **P** associated with a length element ds of a wire carrying a steady current I (see the figure):

• The vector **dB** is perpendicular both to **ds** (which points in the direction of the current) and to the unit vector directed from ds to P.

✓ $d\vec{B} \propto d\vec{s} x \hat{r}$

• The magnitude of **dB** is inversely proportional to $\mathbf{r^2}$, where r is the distance from ds to P.

✓ $d\vec{B} \propto \frac{1}{\vec{r}^2}$

• The magnitude of **dB** is proportional to the **current** (I) and to the magnitude **ds** of the length element ds.

✓ $d\vec{B} \propto I\, d\vec{s}$

• The magnitude of **dB** is proportional to **sin$_\theta$**, where θ is the angle between the vectors **ds** and $\hat{r}$.

✓ $d\vec{B} \propto \sin\theta$

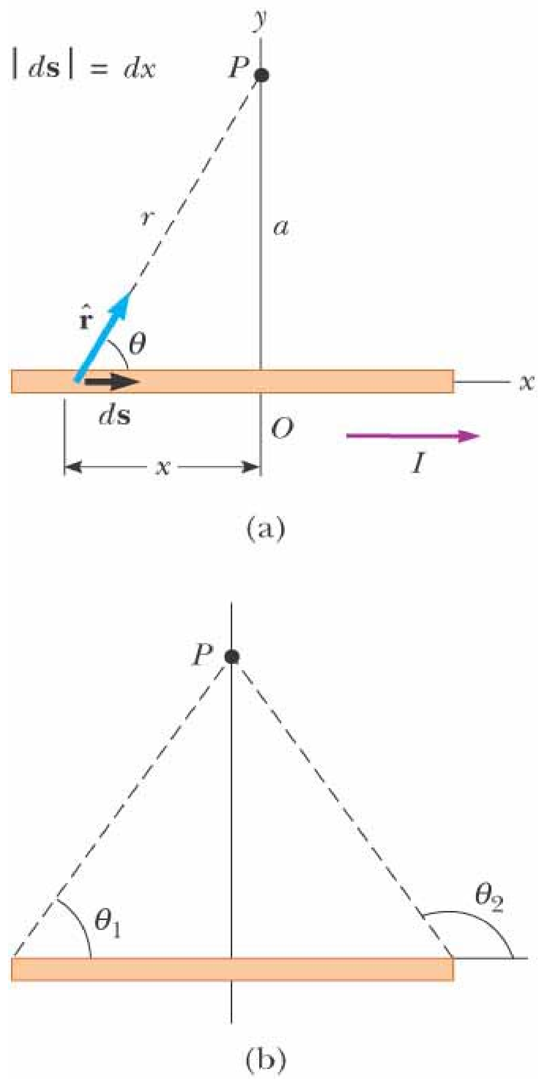
These observations can be summarized in the following equation,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\ ds\ \sin_\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\ d\vec{s}\ x\hat{r}}{r^2} \qquad 29.1$$

Where: $\mu_0 = 4\pi\ x\ 10^{-7}\, Wb/A.m$ and called the permeability of the free space.

To find the total magnetic field B created at some point by a current of finite size, we must sum up contributions from all current elements *Ids that make up the current. That is, we* must evaluate B by integrating Equation 30.1,

$$\vec{B} = \frac{\mu_0\, I}{4\pi}\int \frac{d\vec{s}\ x\ \hat{r}}{r^2}$$

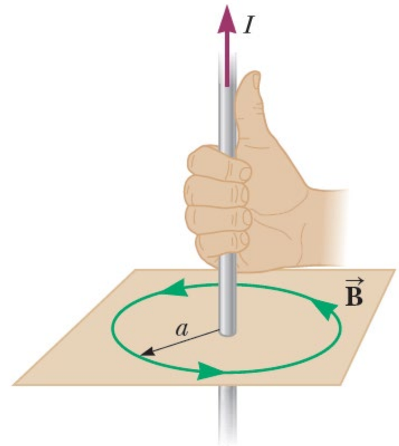


Application of Biot-Savart Law:

Magnetic field surrounding a thin, straight conductor,

$$B = \frac{\mu_0}{2\pi}\frac{I}{a} \qquad 29.2$$

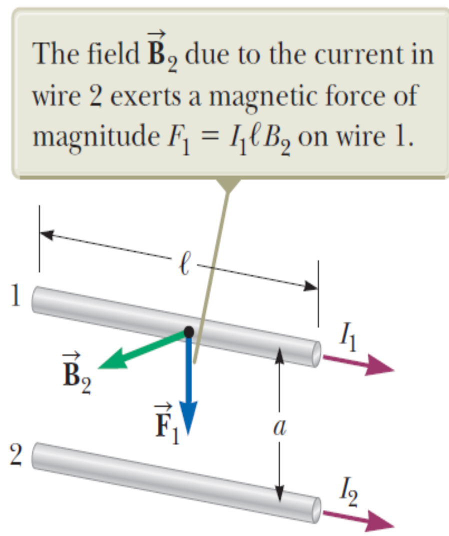
The right-hand rule is used to determine the direction of the magnetic field surrounding a long, straight wire carrying a current. Note that the magnetic field lines form circles around the wire.



29.2 The magnetic force between two parallel conductors:

Consider two long, straight, parallel wires separated by a distance **a** and carrying currents **I_1** and **I_2** in the same direction.

We can determine the force exerted on one wire due to the magnetic field set up by the other wire.



From the figure, the magnetic field on **wire 1** is,

$$B_2 = \frac{\mu_0}{2\pi} . \frac{I_2}{a} \qquad 29.3$$

The magnetic force acting on **wire 1** is

$$F_1 = I_1 l B_2 \qquad 29.4$$

So, one can substitute eq. (29.3) in eq. (29.4) and get,

$$F_1 = I_1 l \left(\frac{\mu_0}{2\pi} . \frac{I_2}{a}\right)$$

$$F_1 = \frac{\mu_0}{2\pi} . \frac{I_1 I_2}{a} l \qquad 29.5$$

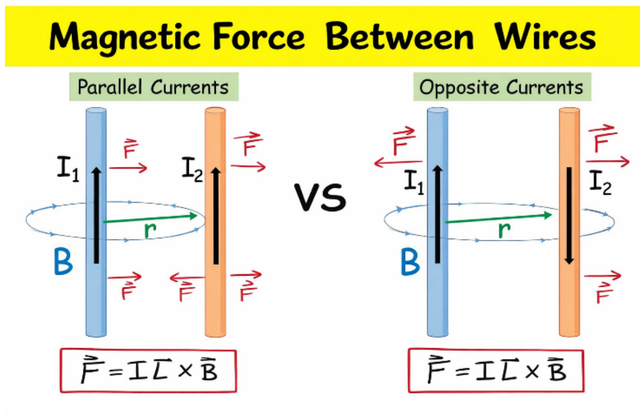
This equation (29.5) can be rewritten in terms of the force per unit length,

$$F = \frac{\mu_0 l}{2\pi} . \frac{I_1 I_2}{a} \qquad 29.6$$

Or, Force pre unit length, $\frac{F}{l} = \frac{\mu_0}{2\pi} . \frac{I_1 I_2}{a}$

Note that:

- **Parallel conductors carrying currents in the same direction attract each other, and parallel conductors carrying currents in opposite directions repel each other.**
- **When the field set up at wire 2 by wire 1 is calculated, the force F_2 acting on wire 2 is found to be equal in magnitude and opposite in direction to F_1.**



Hence, parallel conductors carrying currents in the same direction *attract* each other, and parallel conductors carrying currents in *opposite* directions *repel* each other.

Think about it:

Determine the direction of the magnetic force on each wire.

Examples:

Q1: Calculate the magnitude of the magnetic field at a point 100 cm from a long, thin conductor carrying a current of 1.00 A.

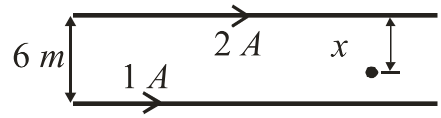
Solution:

For a long straight wire: $B = \mu_0 I / (2\pi a)$. Here a = 100 cm = 1.00 m, I = 1.00 A, and $\mu_0 = 4\pi\times10^{-7}$ T·m/A.

$B = (4\pi\times10^{-7} \times 1.00) / (2\pi \times 1.00) = 2\times10^{-7}$ T.

Answer: 2.0×10^{-7} T.

Q2- What is the distance *x* at which the magnetic field vanishes?



A) 1   B) 2   C) 4   D) 5

Solution:

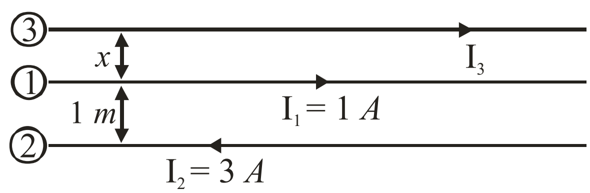
Magnetic fields from the two wires at a point between them are opposite in direction. Let the distance to the top wire be x and to the bottom wire be 6-x.

Set magnitudes equal: $\mu_0(2)/(2\pi x) = \mu_0(1)/(2\pi(6 - x)) \Rightarrow 2/x = 1/(6 - x)$.

Solve: $2(6 - x) = x \Rightarrow 12 - 2x = x \Rightarrow 3x = 12 \Rightarrow x = 4$ m.

Answer: x = 4 m.

Q3: The distance x at which the force acting on wire number 3 is equal to zero is



A) 1.5   B) 1   C) 0.5   D) 0.25

Solution:

Forces between parallel currents: same directions → attraction; opposite → repulsion.

On wire (3): attraction toward wire (1) (downward) with magnitude $F_{31}/L = \mu_0 I_3 I_1 / (2\pi x)$. Repulsion from wire (2) (upward) with magnitude $F_{32}/L = \mu_0 I_3 I_2 / [2\pi (x + 1)]$.

Set $|F_{31}| = |F_{32}|$ for zero net force and cancel common factors $\mu_0 I_3 /(2\pi)$: $I_1/x = I_2/(x + 1)$.

With $I_1 = 1$ A and $I_2 = 3$ A: $1/x = 3/(x + 1) \Rightarrow x + 1 = 3x \Rightarrow 2x = 1 \Rightarrow x = 0.50$ m.

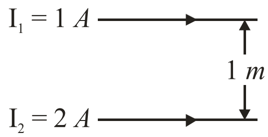
Answer: x = 0.50 m above the wire (1).

Q4- Two long, parallel conductors separated by 10 *cm* carry currents in the same direction, $I_1 = 5$ *A* and $I_2 = 8$ *A*. The force per unit length exerted on each conductor by the other is:

A) 32×10^{-5} B) 16×10^{-5}

C) 8×10^{-5} D) 4×10^{-5}

Q5: The force per unit length upon the wires is:



A) 16×10^{-7} B) 12×10^{-7} C) 8×10^{-7} D) 4×10^{-7}

Q6: The Biot-Savart Law states that the magnetic field dB due to a current element is:

a) Directly proportional to the square of the distance from the element
b) Directly proportional to the current and inversely proportional to the square of the distance
c) Inversely proportional to the current and directly proportional to the distance
d) Directly proportional to the current and inversely proportional to the distance

Q7: The magnetic field around a long, straight current-carrying conductor forms:

a) Straight lines along the wire
b) Concentric circles around the wire
c) A helical pattern along the wire
d) Randomly distributed field lines

Q8: The strength of the magnetic field around a straight conductor:

a) Increases as the distance from the wire increases
b) Is independent of the distance from the wire
c) Decreases as the distance from the wire increases
d) Depends only on the wire's length

Q9: Two long, straight, parallel wires are separated by 20 cm and carry currents of 5 A and 10 A, both in the same direction. At what point along the line joining the two wires is the net magnetic field zero?

a) 6.67 cm from the 5 A wire
b) 10 cm from the 5 A wire
c) 13.33 cm from the 5 A wire
d) Net magnetic field is never zero