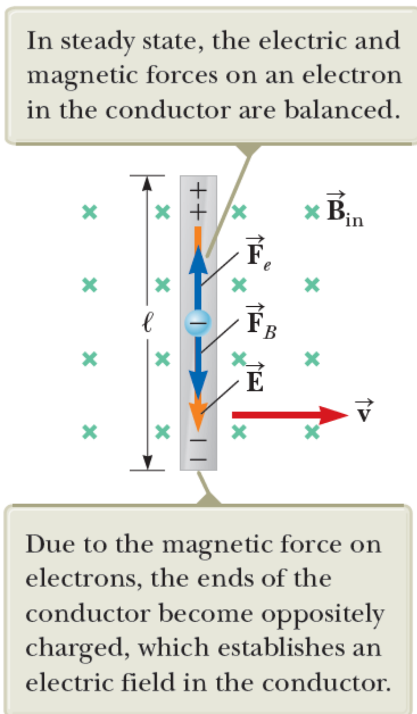
31.2 Motional emf
We analyzed a situation in which a coil of wire was stationary while the magnetic field changed over time. Let's now look at something different!

Suppose a magnetic field is uniform and constant, and we move a conductor through it. We find that an emf is induced in the conductor. We call such an emf a **motional emf.**

I-

A straight electrical conductor of length moving with a velocity **v** through a uniform magnetic field **B** directed perpendicular to **v**. Due to the magnetic force on electrons, the ends of the conductor become oppositely charged. This establishes an electric field in the conductor. **In steady state, the electric and magnetic forces on an electron in the wire are balanced.**



$$F_B = F_E$$

$$qvB = qE$$

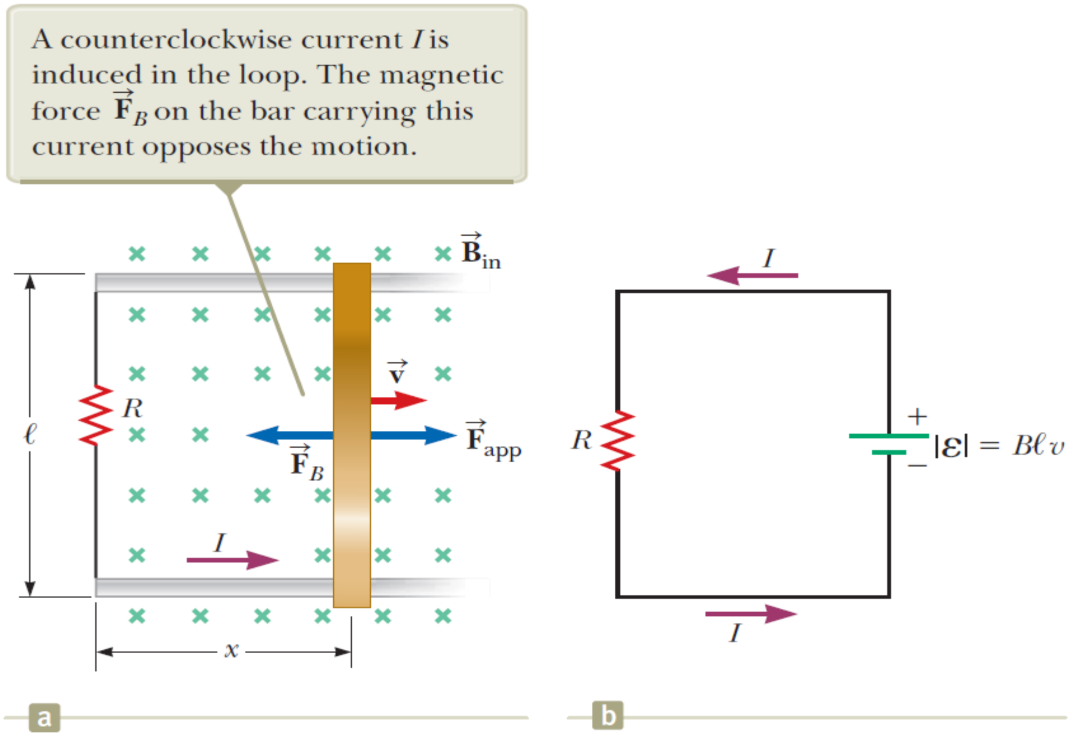
Or, $$E = vB$$

Also, the electric field produced in the conductor is related to the potential difference across the ends of the conductor:

$$\boldsymbol{\Delta V = El = Blv}$$

$==>$ The potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.

II- More general case:



Using Faraday's law, and noting that **x** changes with time at a rate dx/dt = v, we find that the induced motional emf is:

$$\epsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA) = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv$$

If the resistance of the circuit is **R**, the magnitude of the induced current is:

$$I = \frac{|\epsilon|}{R} = \frac{Blv}{R}$$

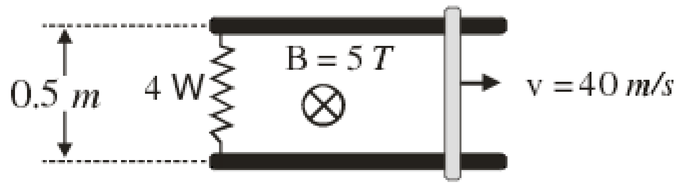
III-

- The applied force does work on the conducting bar.
- The change in energy of the system during some time interval must be equal to the transfer of energy into the system by work.
- The power input is equal to the rate at which energy is delivered to the resistor.

$$P = F_{app.}v = (IlB)v = \left(\frac{Blv}{R}\right)(lB)v = \frac{B^2l^2v^2}{R} = \frac{\epsilon^2}{R}$$

Example 1:

Find the current I in the following circuit.



This problem involves **motional electromotive force (EMF)**, which is the voltage induced across a conductor moving through a uniform magnetic field.

1. Calculate the Motional EMF (ϵ)

The motional EMF induced across a conductor of length L moving at a velocity v perpendicular to a magnetic field B is given by the formula:
ϵ= B L v

Substitute the given values:

- B = 5 T
- L = 0.5 m
- v = 40 m/s

ϵ= 5 x 0.5 x 40

ϵ= 100 V

2. Calculate the Induced Current (I)

The induced current (I) flowing through the circuit is determined using **Ohm's Law**, where the induced EMF ϵ) acts as the voltage source and R is the total resistance of the circuit:
I = ϵ \R

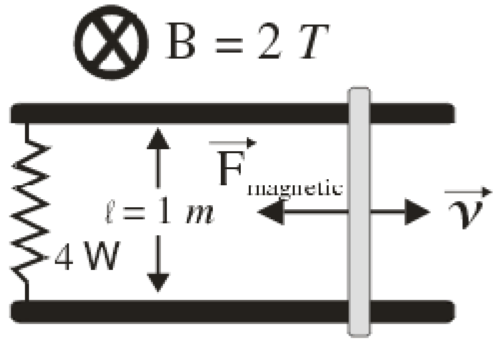
Substitute the calculated EMF and the given resistance:

- ϵ = 100 V
- R = 4 Ω

I = 100 \4

I = 25 A

Example 2:



Calculate the magnetic force ($F_{Magnetic}$) in the following figure, exerted on the bar when it moves to the right at a constant speed of 200 cm/s in a uniform 2.0 T magnetic field directed into the page.

Solution:

Step 1: Compute the motional emf induced in the moving bar:

$\varepsilon = B\,L\,v = (2.0\text{ T})(1.0\text{ m})(2.0\text{ m/s}) = 4.0\text{ V}$

Step 2: Find the current in the circuit:

$I = \varepsilon / R = 4.0\text{ V} / 4.0\ \Omega = 1.000\text{ A}$

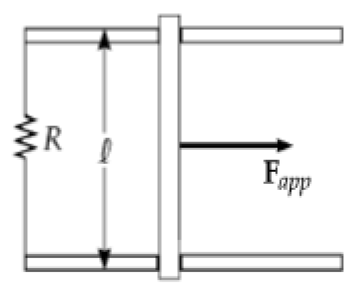
Step 3: Compute the magnetic force on the bar:

$F = B\,I\,L = (2.0\text{ T})(1.000\text{ A})(1.0\text{ m}) = 2.000\text{ N}$

Direction: By Lenz's law, the magnetic force acts to the left, opposing the motion of the bar.

Answer: $F_{magnetic}$ = 2.000 N to the left (opposing motion).

Example 3:



A bar of length *l* =1m moves on two horizontal frictionless rails with a constant speed of 3 m/s in a magnetic field B = 4T directed perpendicularly downward into the paper as shown in the figure. If the applied force required to move the bar to the right is 6 N., find **the resistance R.**

Solution

Step 1: The magnetic force on the bar equals the applied force when moving at constant speed:

$F_{app} = F_{magnetic} = B\,I\,\ell$

Step 2: The induced emf across the bar is given by $\varepsilon = B\,\ell\,v$.

$\varepsilon = (4.0\text{ T})(1.0\text{ m})(3.0\text{ m/s}) = 12.0\text{ V}.$

Step 3: Using Ohm's law, $I = \varepsilon / R$. Substitute into the force equation:

$F_{app} = B (\varepsilon / R) \ell \rightarrow R = B^2 \ell^2 v / F_{app}$.

$R = (4.0^2 \times 1.0^2 \times 3.0) / 6.0 = 8.00\ \Omega$.

Answer: $R = 8.00\ \Omega$.

Concept	Expression	Key Idea
Faraday's Law	$\varepsilon = - d\Phi_B / dt$	EMF arises from a change in magnetic flux
Motional emf	$\varepsilon = B\ell v$	EMF arises from motion in a magnetic field
Lenz's Law	Opposes change	Ensures energy conservation
Power balance	$Fv = I^2R$	Mechanical → Electrical → Thermal