

30.2 Motional emf

We analyzed a situation in which a coil of wire was stationary while the magnetic field changed over time. Let's now look at something different!

Suppose a magnetic field is uniform and constant, and we move a conductor through it. We find that an emf is induced in the conductor. We call such an emf a **motional emf**.

I-

A straight electrical conductor of length ℓ moving with a velocity \mathbf{v} through a uniform magnetic field \mathbf{B} directed perpendicular to \mathbf{v} . Due to the magnetic force on electrons, the ends of the conductor become oppositely charged. This establishes an electric field in the conductor. **In steady state, the electric and magnetic forces on an electron in the wire are balanced.**

$$F_B = F_E$$

$$qvB = qE$$

Or,

$$E = vB$$

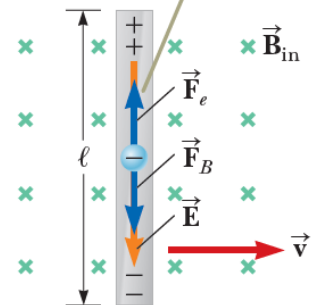
Also, the electric field produced in the conductor is related to the potential difference across the ends of the conductor:

$$\Delta V = El = Blv$$

==> The potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.

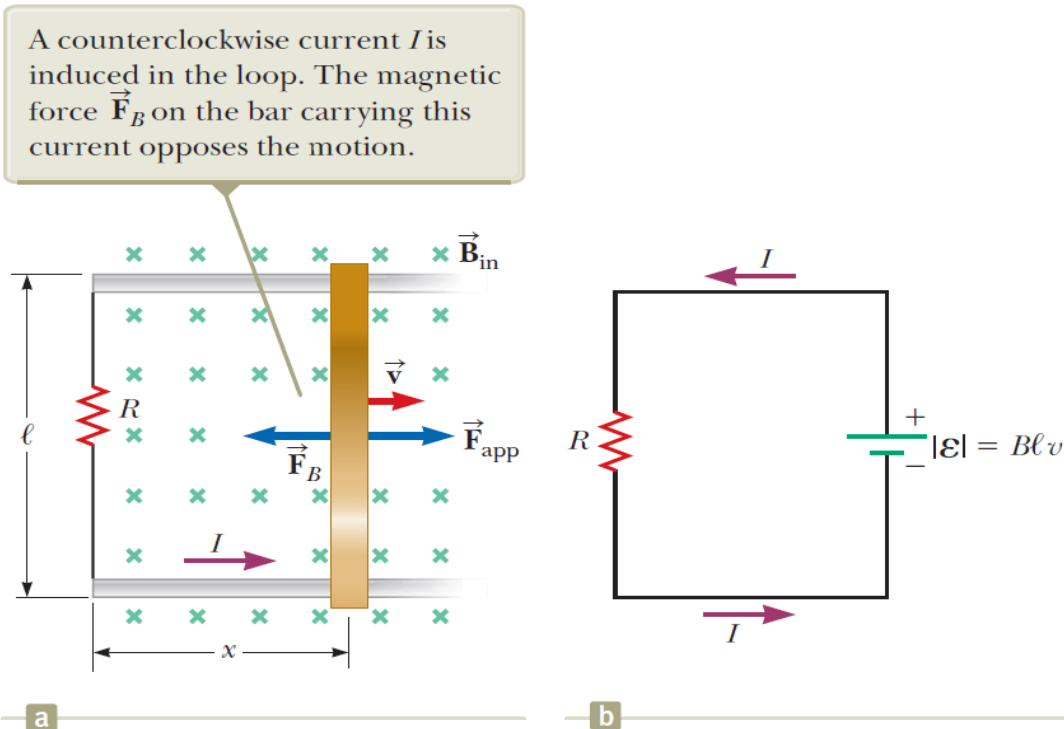
- ✓ Electrons inside the conductor experience a magnetic force → they move → charges separate → voltage appears.

In steady state, the electric and magnetic forces on an electron in the conductor are balanced.



Due to the magnetic force on electrons, the ends of the conductor become oppositely charged, which establishes an electric field in the conductor.

II- More general case:



Using Faraday's law, and noting that x changes with time at a rate $dx/dt = v$, we find that the induced motional emf is:

$$\epsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA) = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv$$

Why do we see a minus sign?

Remember: The induced current always opposes the motion that produces it.

If the resistance of the circuit is R , the magnitude of the induced current is:

$$I = \frac{|\epsilon|}{R} = \frac{Blv}{R}$$

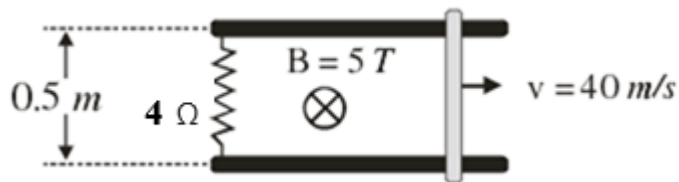
III-

- The applied force does work on the conducting bar.
- The change in energy of the system during some time interval must be equal to the transfer of energy into the system by work.
- The power input is equal to the rate at which energy is delivered to the resistor.

$$P = F_{app} \cdot v = (IlB)v = \left(\frac{Blv}{R}\right)(lB)v = \frac{B^2 l^2 v^2}{R} = \frac{\epsilon^2}{R}$$

Example 1:

Find the current I in the following circuit.



This problem involves **motional electromotive force (EMF)**, which is the voltage induced across a conductor moving through a uniform magnetic field.

1. Calculate the Motional EMF (ϵ)

The motional EMF induced across a conductor of length L moving at a velocity v perpendicular to a magnetic field B is given by the formula:

$$\epsilon = B L v$$

Substitute the given values:

- $B = 5 \text{ T}$
- $L = 0.5 \text{ m}$
- $v = 40 \text{ m/s}$

$$\epsilon = 5 \times 0.5 \times 40$$

$$\epsilon = 100 \text{ V}$$

2. Calculate the Induced Current (I)

The induced current (I) flowing through the circuit is determined using **Ohm's Law**, where the induced EMF (ϵ) acts as the voltage source, and R is the total resistance of the circuit:

$$I = \epsilon / R$$

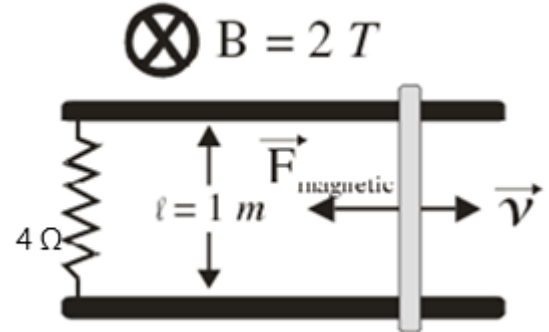
Substitute the calculated EMF and the given resistance:

- $\epsilon = 100 \text{ V}$
 - $R = 4 \Omega$
- $$I = \epsilon / R = 100 / 4$$

$$I = 25 \text{ A}$$

Example 2:

Calculate the magnetic force (F_{Magnetic}) in the following figure, exerted on the bar when it moves to the right at a constant speed of 200 cm/s in a uniform 2.0 T magnetic field directed into the page.



Solution:

Step 1: Compute the motional emf induced in the moving bar:

$$\varepsilon = B L v = (2.0 \text{ T})(1.0 \text{ m})(2.0 \text{ m/s}) = 4.0 \text{ V}$$

Step 2: Find the current in the circuit:

$$I = \varepsilon / R = 4.0 \text{ V} / 4.0 \Omega = 1.000 \text{ A}$$

Step 3: Compute the magnetic force on the bar:

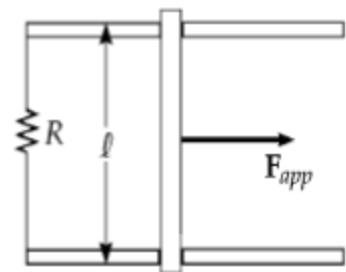
$$F = B I L = (2.0 \text{ T})(1.000 \text{ A})(1.0 \text{ m}) = 2.000 \text{ N}$$

Direction: By Lenz's law, the magnetic force acts to the left, opposing the motion of the bar.

Answer: $F_{\text{magnetic}} = 2.000 \text{ N}$ to the left (opposing motion).

Example 3:

A bar of length $l = 1 \text{ m}$ moves on two horizontal frictionless rails with a constant speed of 3 m/s in a magnetic field $B = 4 \text{ T}$ directed perpendicularly downward into the paper as shown in the figure. If the applied force required to move the bar to the right is 6 N., find **the resistance R**.



Solution

Step 1: The magnetic force on the bar equals the applied force when moving at constant speed:

$$F_{\text{app}} = F_{\text{magnetic}} = B I l$$

Step 2: The induced emf across the bar is given by $\varepsilon = B l v$.

$$\varepsilon = (4.0 \text{ T})(1.0 \text{ m})(3.0 \text{ m/s}) = 12.0 \text{ V}.$$

Step 3: Using Ohm's law, $I = \varepsilon / R$. Substitute into the force equation:

$$F_{\text{app}} = B (\varepsilon / R) \ell \rightarrow R = B^2 \ell^2 v / F_{\text{app}}.$$

$$R = (4.0^2 \times 1.0^2 \times 3.0) / 6.0 = 8.00 \ \Omega.$$

Answer: $R = 8.00 \ \Omega$.

Example 4:

If the speed of the rod is doubled, what happens?

- A) EMF doubles, current doubles, required force doubles
- B) EMF doubles, current doubles, required force quadruples
- C) EMF doubles, current remains constant, force doubles
- D) EMF remains constant, current doubles

Example 5:

A rod moves to the right in a magnetic field into the page.

If the velocity reverses direction, what happens?

- A) Current direction remains the same
- B) Current direction reverses
- C) No current is induced
- D) Magnetic force disappears

👉 “What if BOTH velocity and magnetic field reverse simultaneously?”

Answer:

- ✓ Current direction remains the same

Concept	Expression	Key Idea
Faraday's Law	$\varepsilon = - d\Phi_B / dt$	EMF arises from a change in magnetic flux

Motional emf	$\varepsilon = B\ell v$	EMF arises from motion in a magnetic field
Lenz's Law	Opposes change	Ensures energy conservation
Power balance	$Fv = I^2R$	Mechanical \rightarrow Electrical \rightarrow Thermal