1

Lecture (9)

4.2.2 parametric Distribution Families

p.52

Defn

A parametric distribution family is a set of parametric distributions that are related in some meaningful way.

Ex 4.3 p.52

Demonstrate that the transformed beta family as defined in Appendix A is a parametric distribution family

Ans:

For $X \sim$ Transformed beta $-\alpha, \theta, \gamma, \tau$ (generalized beta) p.463

$$f(x) = \frac{\Gamma(\alpha+\tau)}{\Gamma(\alpha)\Gamma(\tau)} \cdot \frac{\gamma (x/\theta)^{\gamma\tau}}{x[1+(x/\theta)^{\gamma}]^{\alpha+\tau}} \quad (*)$$

At $\gamma = \tau = 1$ in $(*)$

$$(*) \Rightarrow f(x) = \frac{\Gamma(\alpha+1)(x/\theta)}{\Gamma(\alpha)\Gamma(1)\, x[1+(x/\theta)]^{\alpha+1}}$$

$$= \frac{\alpha!\,(\cancel{x}/\theta)}{(\alpha-1)!\,\cancel{x}[1+(x/\theta)]^{\alpha+1}}$$

$$= \frac{\alpha}{\theta} \Big/ \left(\frac{x+\theta}{\theta}\right)^{\alpha+1}$$

$$\therefore f(x) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}},$$ which is a pareto prob. density fn (1)

See p. 465

, At $\tau = 1$, $\gamma = \alpha$ in $(*)$

$$(*) \Rightarrow f(x) = \frac{\Gamma(\alpha+1)\,\alpha\,(x/\theta)^{\alpha}}{\Gamma(\alpha)\Gamma(1)x[1+(x/\theta)^{\alpha}]^{\alpha+1}}$$

$$f(x) = \frac{\alpha^2 (x/\theta)^{\alpha}}{x[1+(x/\theta)^{\alpha}]^{\alpha+1}},$$ which is called paralogistic p.d.f

See p.466

(2)

We can deduce from $(*)$, (1) and (2) that the transformed beta distn is a parametric distn family #