Lecture 7

Integrals

~ Definite and Indefinite Integrals ~

Theorem

$$\frac{d}{dx}\int_{k(x)}^{g(x)} f(t)\,dt = f(g(x))\,g'(x) - f(k(x))\,k'(x)$$

EX 1

Find each of the following

1) $\frac{d}{dx}\int_{3x}^{x^2} (t^3+1)^{10}\,dt$

Ans:

$$\frac{d}{dx}\int_{3x}^{x^2} (t^3+1)^{10}\,dt$$

$$= \left[(x^2)^3+1\right]^{10}\cdot 2x - \left[(3x)^3+1\right]^{10}\cdot 3$$

$$= 2x(x^6+1)^{10} - 3(27x^3+1)^{10}$$

2) $\frac{d}{dx}\int_{6x-1}^{3} \sqrt{4t+9}\,dt$

$$= 0 - \sqrt{4(6x-1)+9}\,(6)$$

$$= -6\sqrt{24x-4+9}$$

$$= -6\sqrt{24x+5}$$

H.W

pb 1 Find each of the following.

a) $\frac{d}{dx}\int_{2}^{x^4} \frac{t}{\sqrt{t^3+2}}\,dt$

Ans: $\frac{4x^7}{\sqrt{x^{12}+2}}$

b) $\frac{d}{dx}\int_{0}^{3} \sqrt{x^2+16}\,dx$

Ans: 0

c) $\frac{d}{dx}\int_{1}^{x^2} \cos t\,dt$

Ans: $2x\cos x^2$

d) $\frac{d}{dx}\int_{1+3x^2}^{4} \frac{1}{2+e^t}\,dt$

Ans: $-\frac{6x}{2+e^{(1+3x^2)}}$

pb 2

Find the value of c that satisfies the mean value theorem of $f(x)=\sqrt{x+1}$ on $[-1,8]$, then find the average value of f on $[-1,8]$.

Ans: (i) $c=3$ (ii) $av(f)=2$

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EX ② Evaluate the following Integrals.

(a) $\int \sqrt{2x+1}\, dx$

Ans:

$I = \frac{1}{2}\int \sqrt{2x+1}\; 2\, dx$

$I = \frac{1}{2}\, \frac{(2x+1)^{3/2}}{3/2} + C$

$\therefore I = \frac{1}{3}(2x+1)^{3/2} + C$

(b) $\int x\sqrt{2x+1}\, dx$

Ans: let $u = 2x+1 \Rightarrow x = \frac{u-1}{2}$

$\Rightarrow dx = \frac{1}{2}du$

$I = \frac{1}{2}\int \left(\frac{u-1}{2}\right)\sqrt{u}\, du$

$I = \frac{1}{4}\int (u^{3/2} - u^{1/2})\, du$

$I = \frac{1}{4}\left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right] + C$

$\therefore I = \frac{1}{10}(2x+1)^{5/2} - \frac{1}{6}(2x+1)^{3/2} + C$

(c) $\int \cos(7\theta+3)\, d\theta$

Ans:

$I = \frac{1}{7}\int \cos(7\theta+3)\, 7\, d\theta$

$I = \frac{1}{7}\sin(7\theta+3) + C$

OR let $u = 7\theta+3 \Rightarrow du = 7d\theta$

$\Rightarrow d\theta = \frac{du}{7}$

$I = \frac{1}{7}\int \cos u\, du$

$= \frac{1}{7}\sin u + C$

$= \frac{1}{7}\sin(7\theta+3) + C$

(d) $\int \sin^2 x\, dx$

Ans:

$\because \sin^2 x = \frac{1-\cos 2x}{2}$

$I = \frac{1}{2}\int (1 - \cos 2x)\, dx$

$I = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + C$

$I = \frac{1}{2}x - \frac{\sin 2x}{4} + C$

$\int \cos ax\, dx = \frac{\sin ax}{a} + C$

$\int \sin ax\, dx = \frac{-\cos ax}{a} + C$

$, a \in \mathbb{R}$

H.W

pb3 Evaluate each of the following Integral.

(a) $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\, dx$

Hint: $u = x^3+1$ Ans: $\frac{4\sqrt{2}}{3}$

(b) $\int \sec^2(5x+1)\, dx$

Ans: $I = \frac{1}{5}\tan(5x+1) + C$

(c) $\int \cos^2 x\, dx$

Hint: $\cos^2 x = \frac{1+\cos 2x}{2}$

Ans:

$I = \frac{x}{2} + \frac{\sin 2x}{4} + C$