Chapter 6
Some Continuous Probability Distributions:

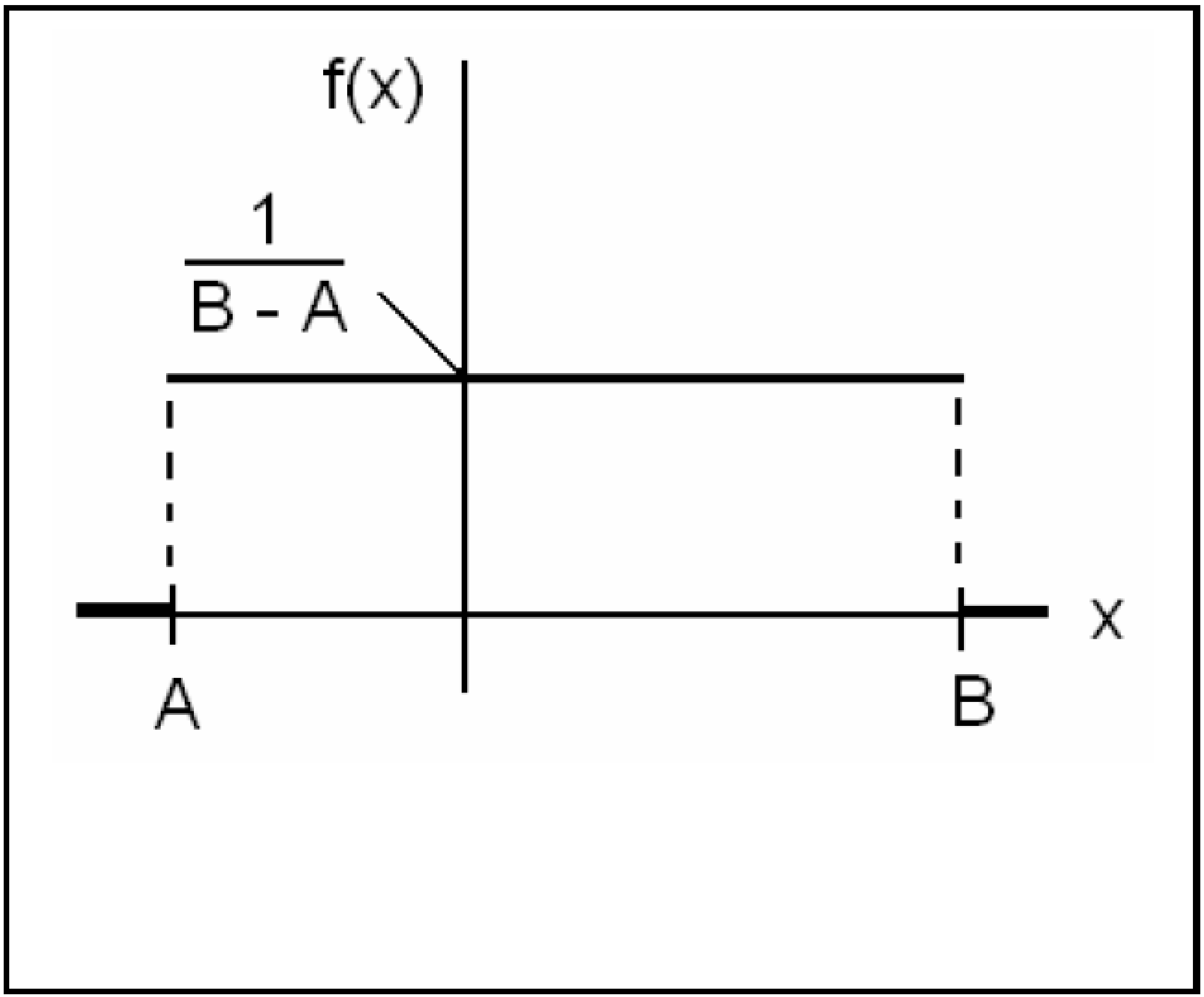
Continuous Uniform distribution

(Rectangular Distribution)

The probability density function of the continuous uniform random variable X on the interval [*A*, *B*] is given by:

$$f(x) = f(x; A, B) = \begin{cases} \dfrac{1}{B - A} & ; \; A \le x \le B \\ 0 & ; \; elsewhere \end{cases}$$

We write X~Uniform(A,B).


f(x)
1
B - A
x
A
B

Theorem

The mean and the variance of the continuous uniform distribution on the interval [*A*, *B*] are:

$$\mu = \frac{A+B}{2}$$

$$\sigma^2 = \frac{(B-A)^2}{12}$$

Example

Suppose that, for a certain company, the conference time, X, has a uniform distribution on the interval [0,4] (hours).

(a) What is the probability density function of X?

(b) What is the probability that any conference lasts at least 3 hours?

Solution:

$$\text{(a)}\ \ f(x)=f(x;0,4)=\begin{cases}\dfrac{1}{4} & ;\ 0\le x\le 4\\ 0 & ;\ elsewhere\end{cases}$$

$$\text{(b) P}(X\geq 3)=\int_3^4 f(x)\,dx=\int_3^4 \frac{1}{4}\,dx=\frac{1}{4}$$

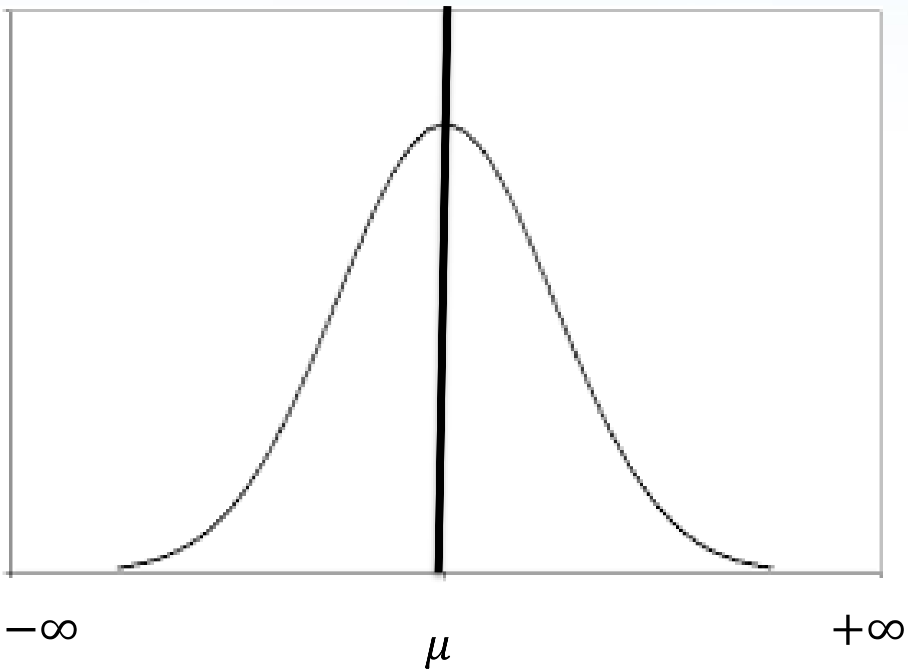
Normal Distribution (Gaussian distribution)

- The normal distribution is one of the most important continuous distributions.
- Many measurable characteristics are normally or approximately normally distributed, such as, height and weight.
- The graph of the probability density function (pdf) of a normal distribution, called the normal curve, is a bell-shaped curve.

The normal distribution characteristics

1) $-\infty < X < \infty$

2) The density function of X , $f(x)$, has a bell-shaped curve



$mean = \mu$

$Varianve = \sigma^2$

3) The highest point of the curve of $f(x)$ at the mean μ

The curve of $f(x)$ is symmetric about the mean μ.

$$\mu = \text{mean} = \text{mode} = \text{median}$$

4) The normal distribution depends on two parameters:

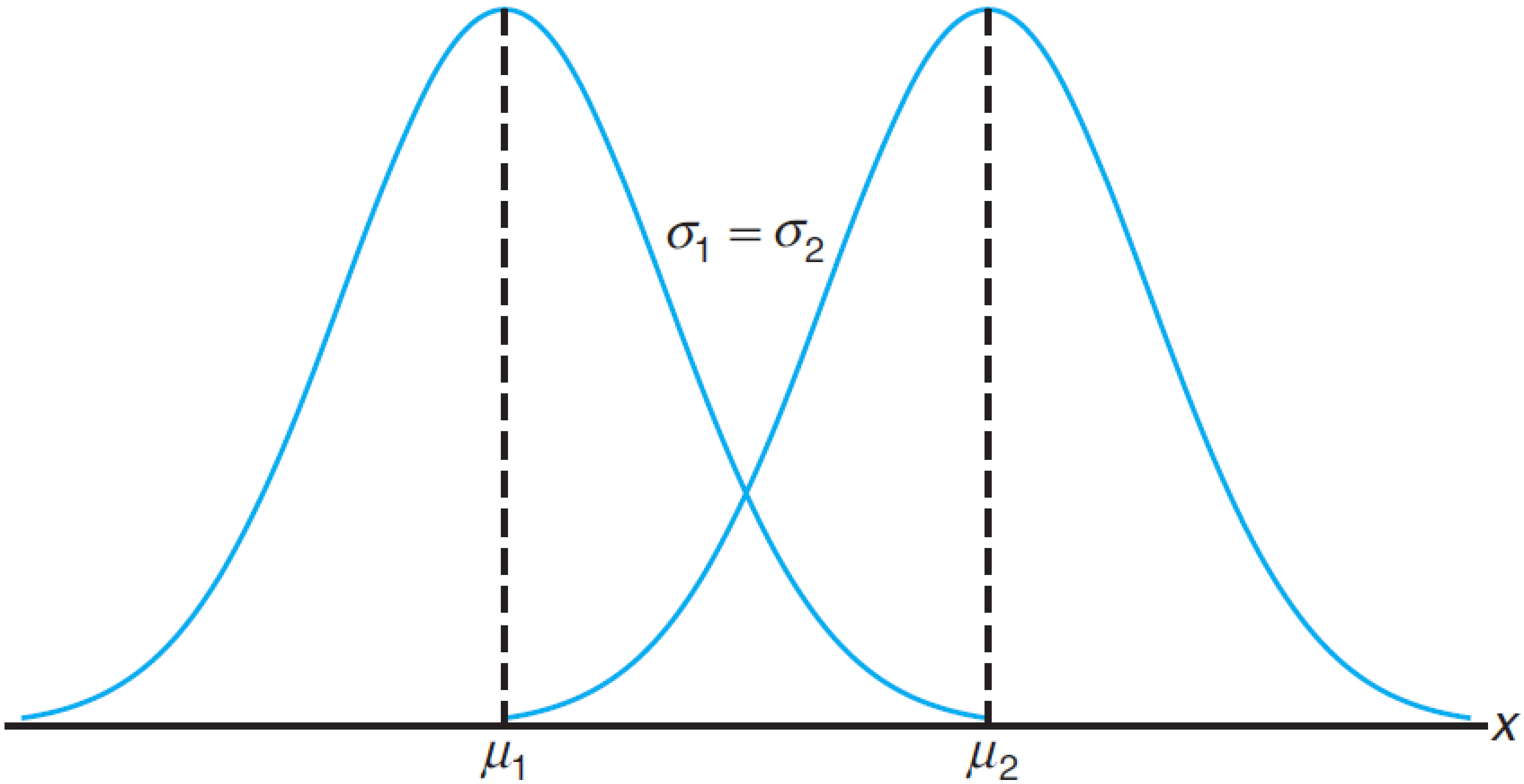
$$\text{mean} = \mu \text{ and variance} = \sigma^2$$

(5) If the r.v. X is normally distributed with mean μ and variance σ^2, we write:

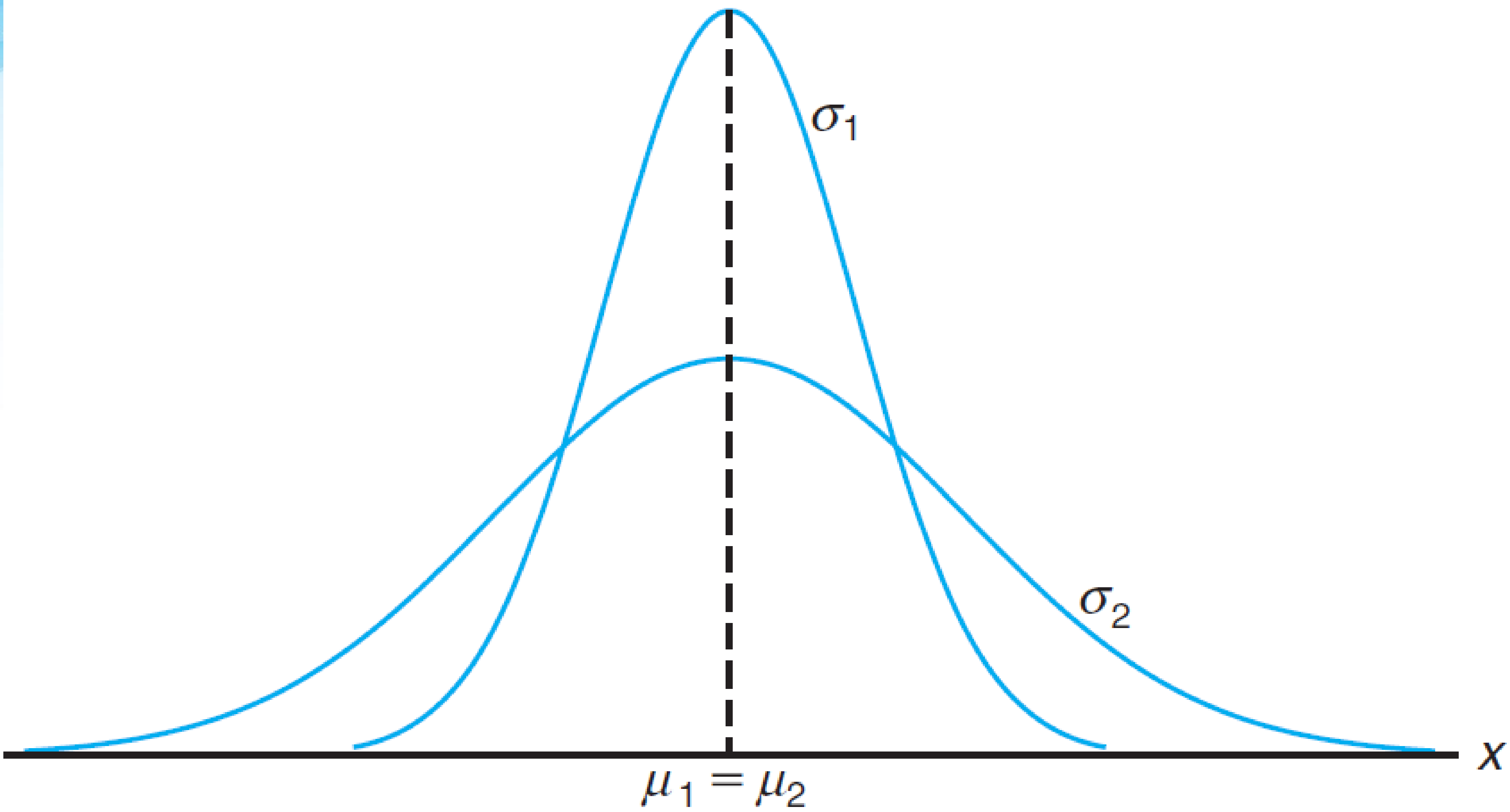
$$X \sim \text{Normal } (\mu, \sigma^2) \text{or } X \sim \text{N}(\mu, \sigma^2)$$

(6) The location of the normal distribution depends on μ

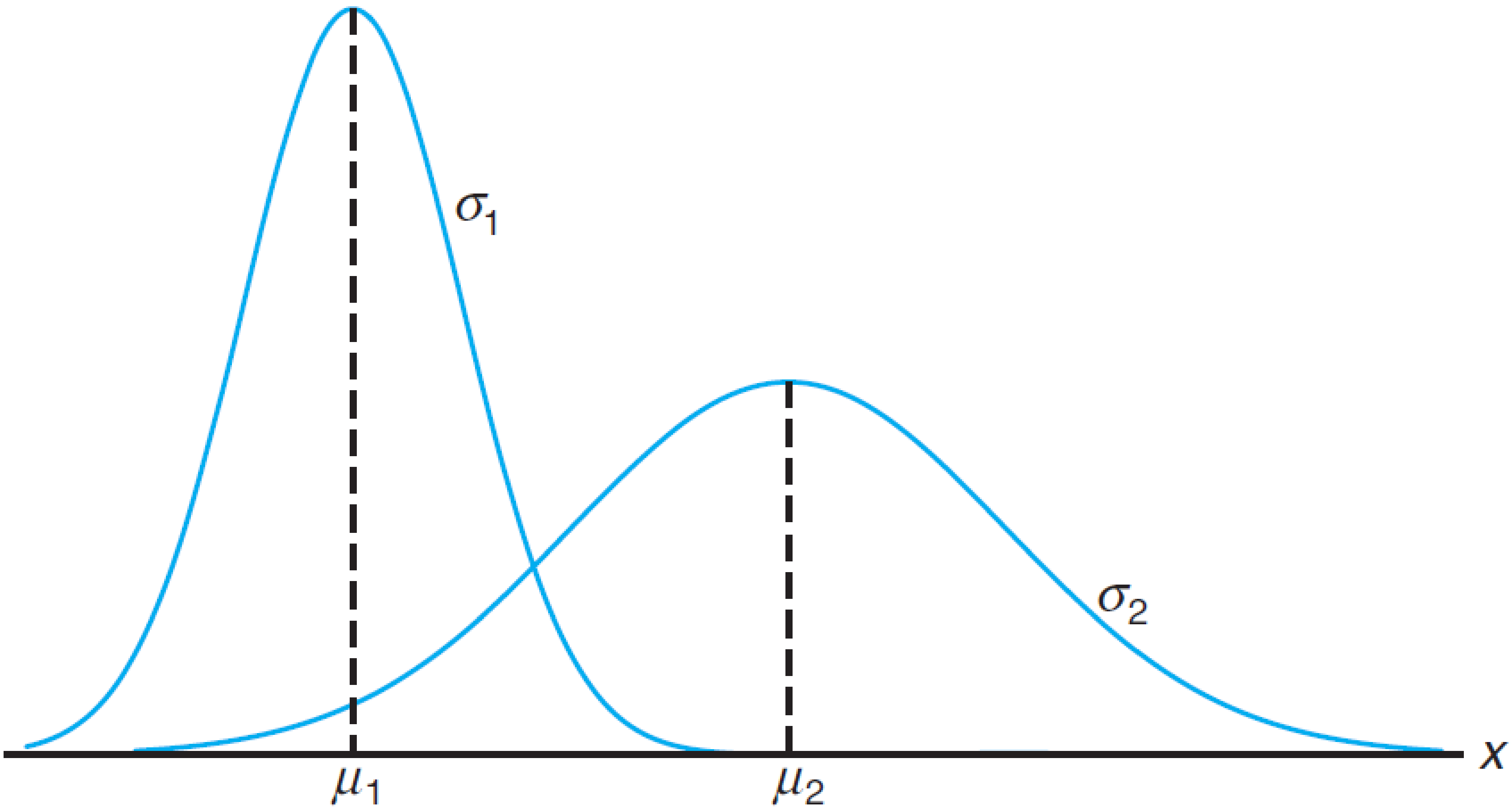
The shape of the normal distribution depends on σ^2



Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$.



Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$.



Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$.

❑ The pdf of $X \sim Normal(\mu, \sigma)$ is given by:

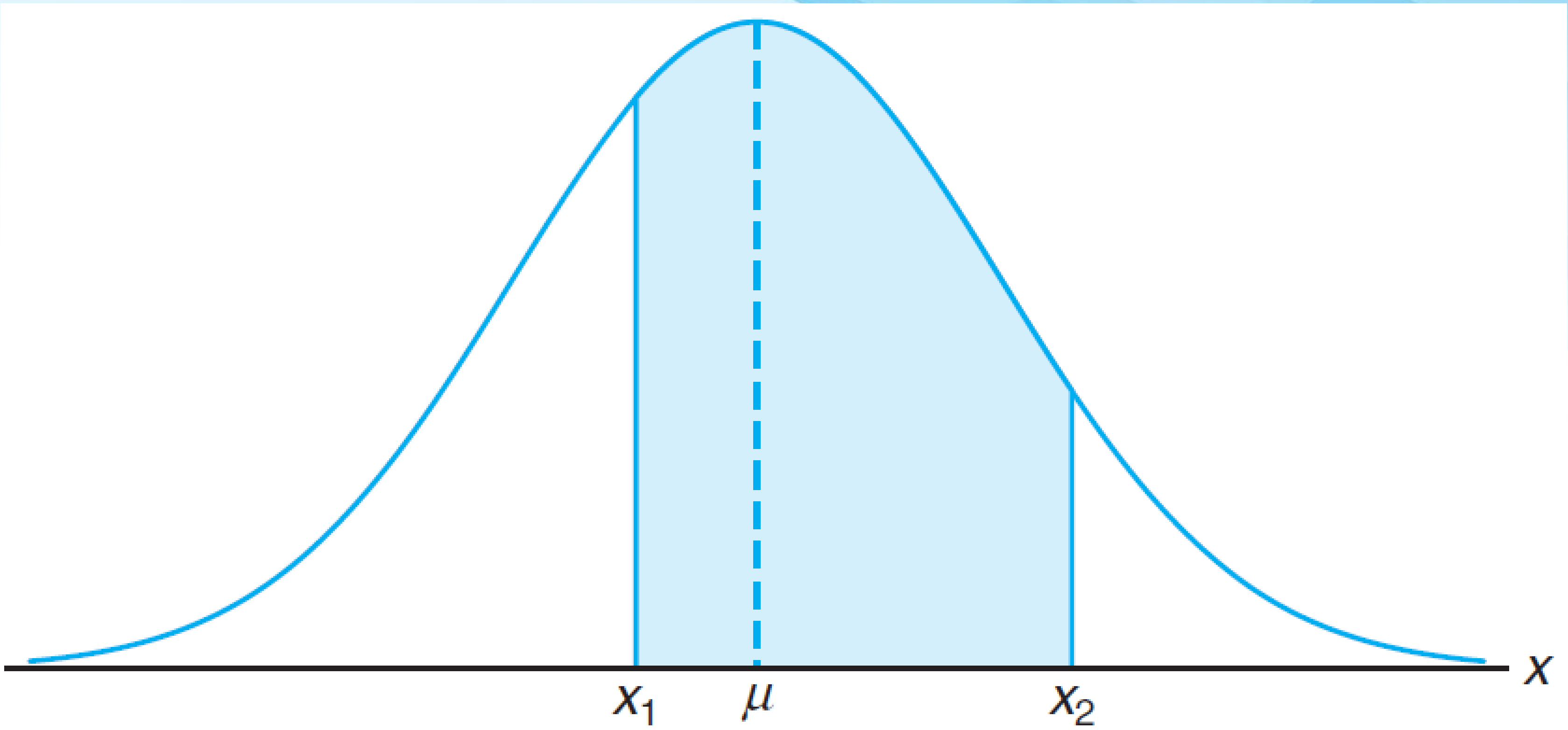
$$f(x) = n(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; \begin{cases} -\infty < x < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{cases}$$

where $\pi = 3.14159\ldots$ and $e = 2.71828\ldots$.

Some properties of the normal curve $f(x)$ of $N(\mu, \sigma)$:

1. $f(x)$ is symmetric about the mean μ.
2. $f(x)$ has two points of inflection at $x = \mu \pm \sigma$.
3. The total area under the curve of $f(x)$ =1.
4. The highest point of the curve of $f(x)$ at the mean μ.

Areas Under the Normal Curve



$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} n(x;\mu,\sigma)\, dx = \frac{1}{\sqrt{2\pi}\sigma}\int_{x_1}^{x_2} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx$$

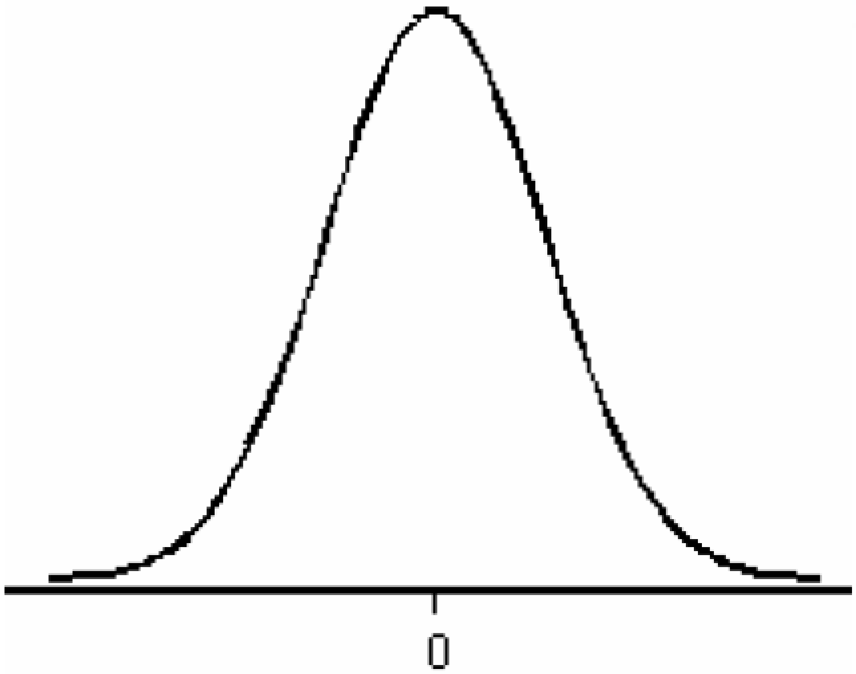
Areas Under the Normal Curve

The Standard Normal Distribution:

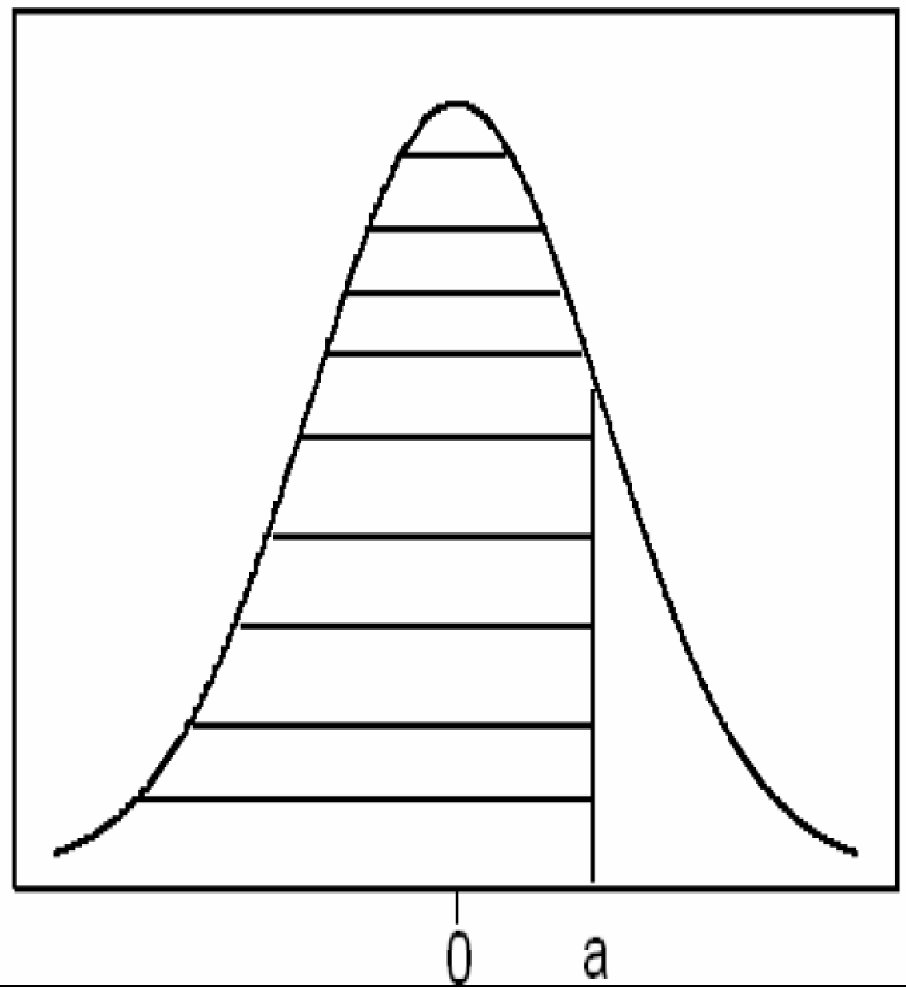
- The normal distribution with mean $\mu=0$ and variance $\sigma^2=1$ is called the standard normal distribution and is denoted by Normal(0,1) or N(0,1). If the random variable Z has the standard normal distribution, we write Z~Normal(0,1) or Z~N(0,1).

- The pdf of Z~N(0,1) is given by:

$$f(z) = n(z;0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$



- The standard normal distribution, $Z \sim N(0,1)$, is very important because probabilities of any normal distribution can be calculated from the probabilities of the standard normal distribution.
- Probabilities of the standard normal distribution $Z \sim N(0,1)$ of the form $P(Z \leq a)$ are tabulated.



$$P(Z \le a) = \int_{-\infty}^{a} f(z)\,dz$$

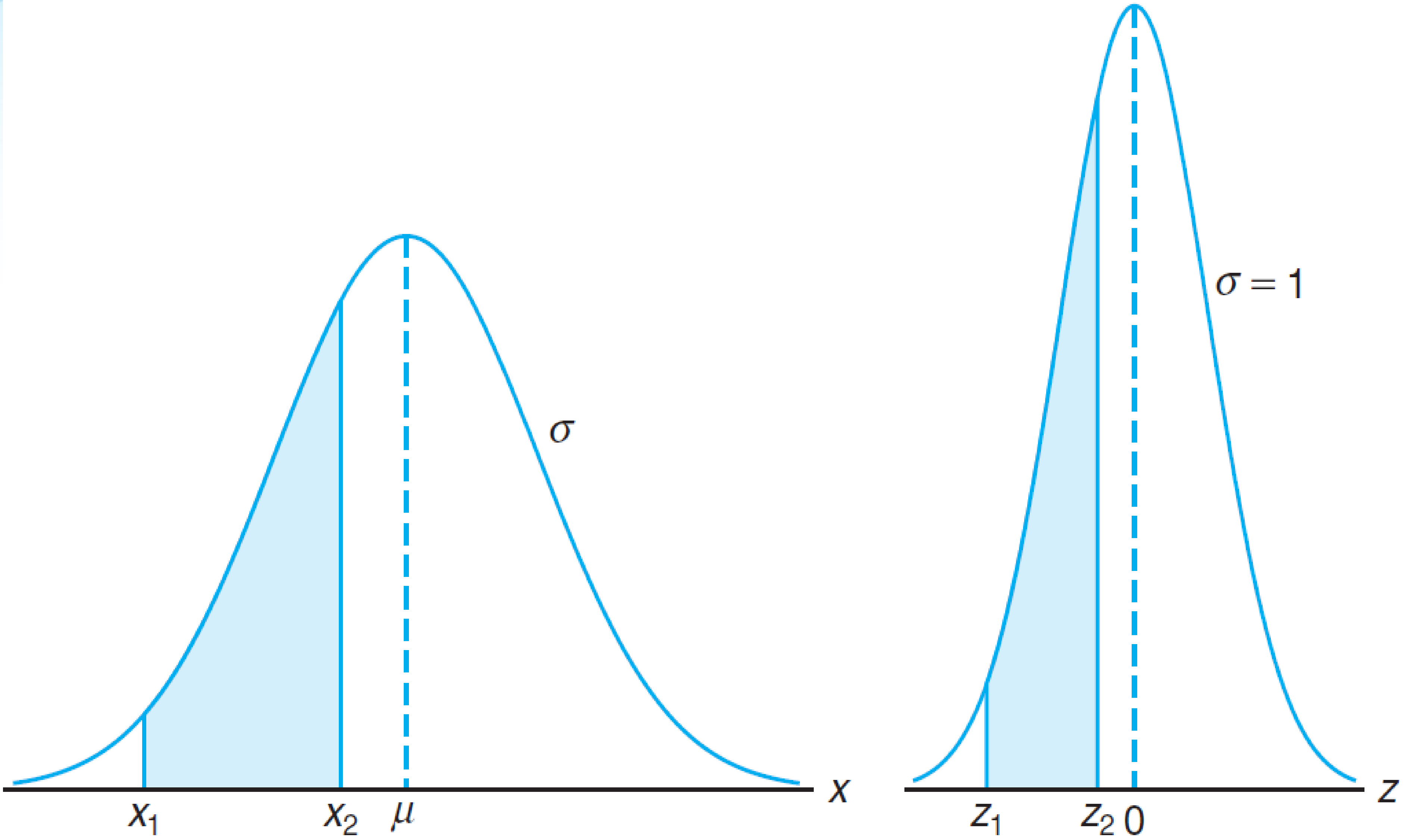
$$= \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz$$

= from the table

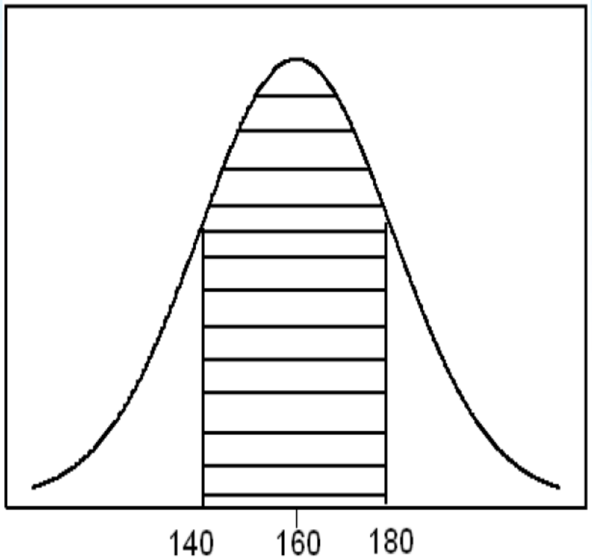
- We can transfer any normal distribution X~N(μ,σ) to the standard normal distribution, Z~N(0,1) by using the following result.

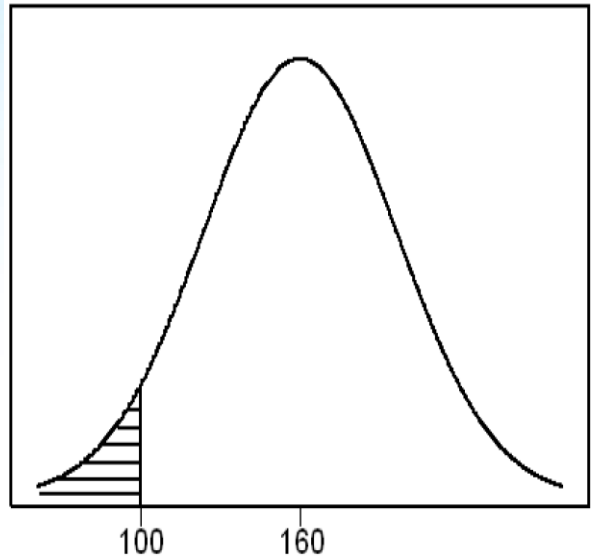
Result: If X~N(μ,σ), then $Z = \dfrac{X-\mu}{\sigma}$~N(0,1).

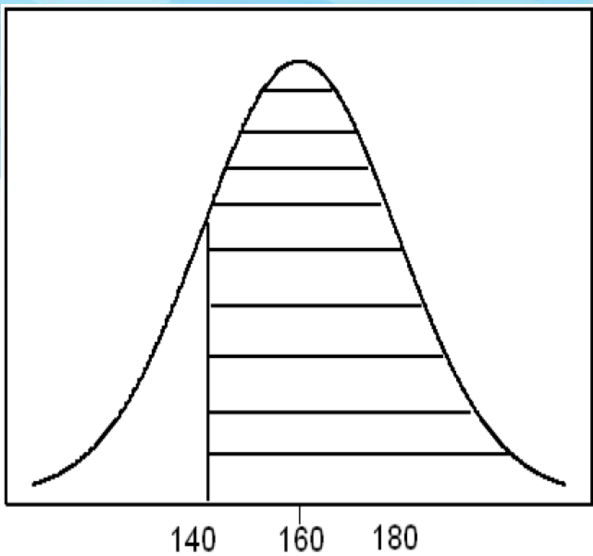
The original and transformed normal distributions.



(ii) Probability of an interval event is given by the area under the curve of $f(x)$ and above that interval.







$$P(140<X<180)=\int_{140}^{180} f_X(x)\,dx \qquad P(X<100)=\int_{-\infty}^{100} f_X(x)\,dx \qquad P(X>140)=\int_{140}^{\infty} f_X(x)\,dx$$

$$area = P(a \le X \le b) = \int_a^b f(x)dx$$

$$area = P(X \le a) = \int_{-\infty}^{a} f(x)dx$$

$$area = P(X \ge b) = \int_b^{\infty} f(x)\,dx$$

Note: Probabilities of Z~N(0,1):

If X is continuous random variable then:

(i) $P(X = x) = 0$ for any x

(ii) $P(X \leq a) = P(X < a)$

(iii) $P(X \geq b) = P(X > b)$

(iv) $P(a \leq X \leq b) = P(a \leq X < b) = P(a < X \leq b) = P(a < X < b)$

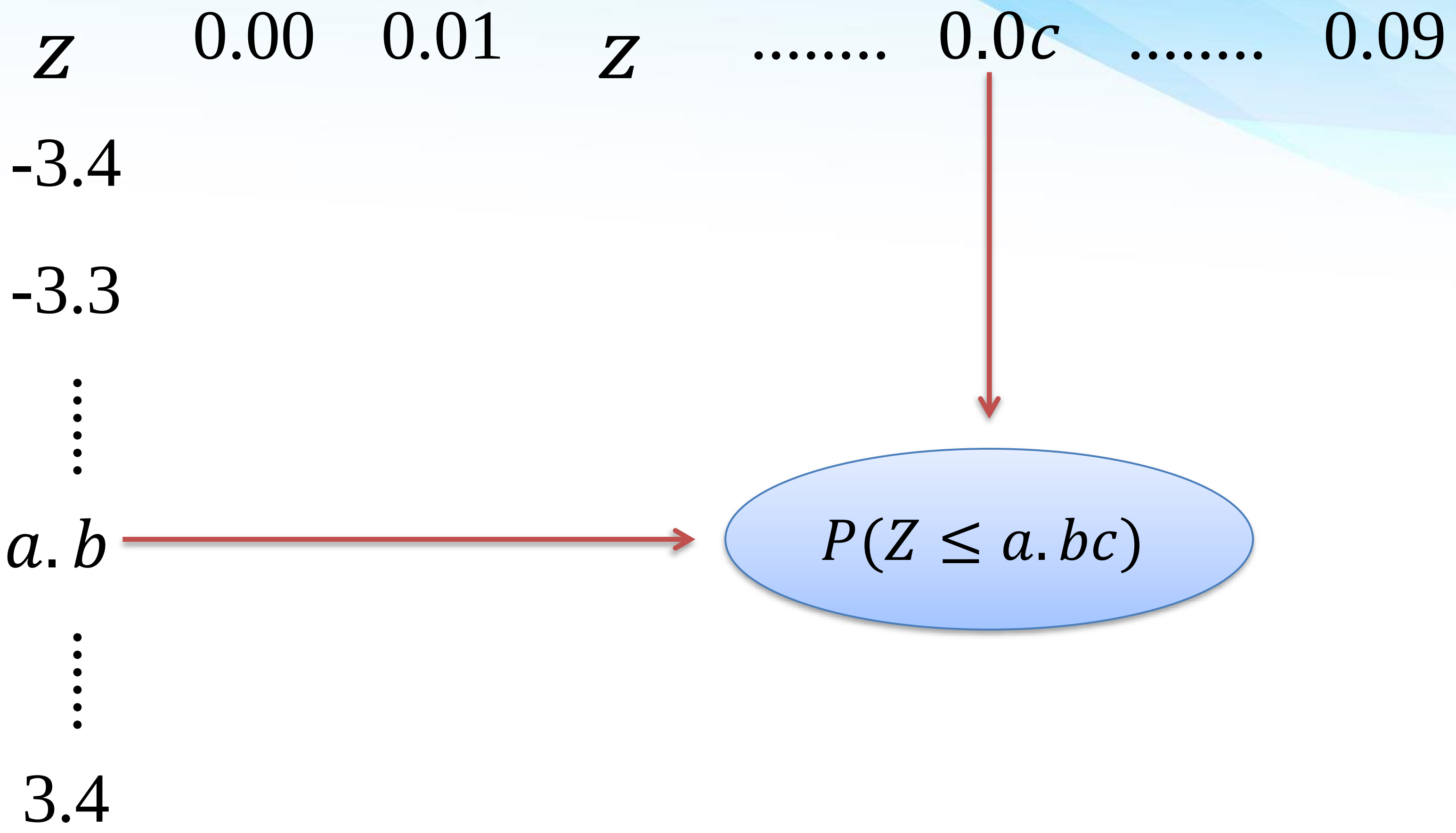
(v) $P(X \leq x) =$ cumulative probability

(vi) $P(X \geq a) = 1 - P(X < a) = 1 - P(X \leq a)$

(vii) $P(a \leq X \leq b) = P(X \leq b) - P(X \leq a)$

Finding $P(Z \leq z)$ from the table

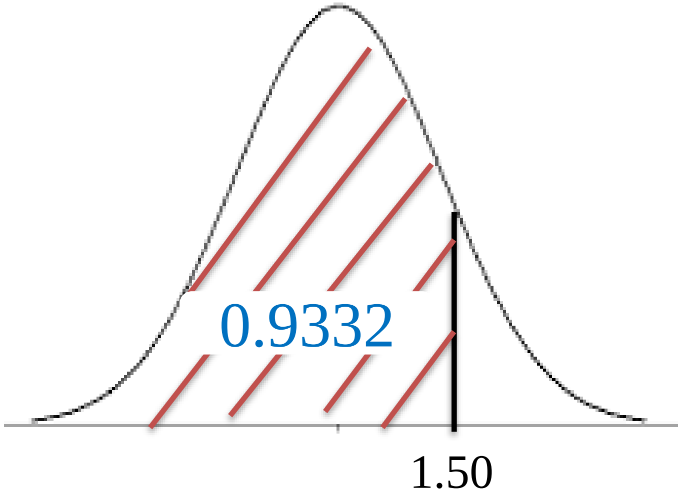
consider that the value of z is rounded to 2 decimal places as $z = a.bc$



Example(1): $Z \sim N(0,1)$

1) $P(Z \leq 1.50)$

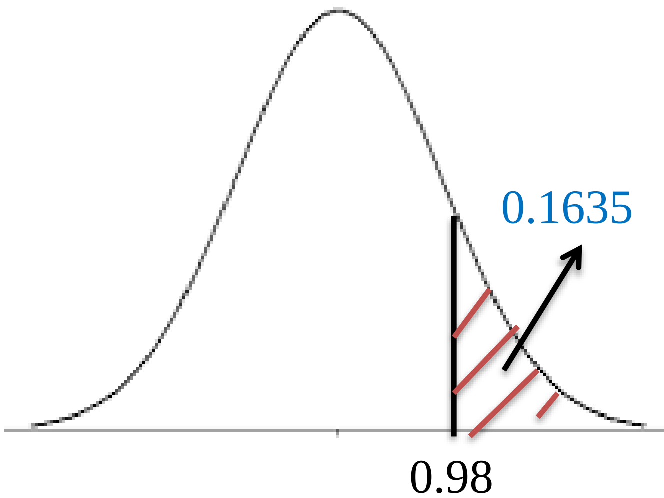
z	0.00	0.01	0.02	....	0.09
-3.4					
-3.3					
⋮					
1.5 →	0.9332				
⋮					
3.4					



Example(2): $Z \sim N(0,1)$

2) $P(Z \geq 0.98) = 1 - P(Z \leq 0.98) = 1 - 0.8365 = 0.1635$

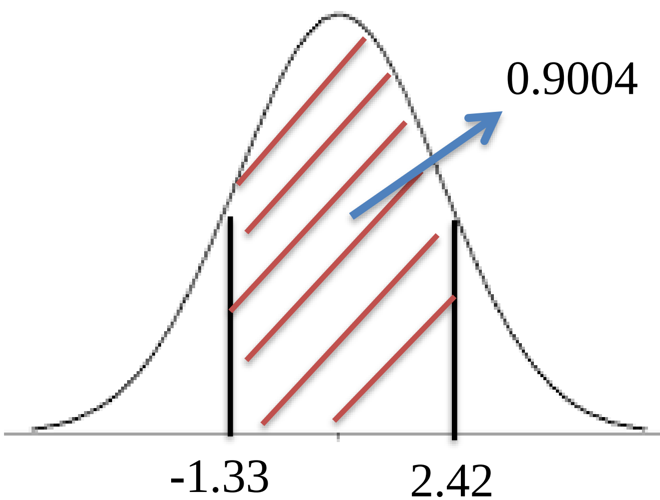
z	0.00	0.01	....	0.08	0.09
-3.4					
-3.3					
⋮					
0.9				0.8365	
⋮					
3.4					



Example (3): $Z \sim N(0,1)$

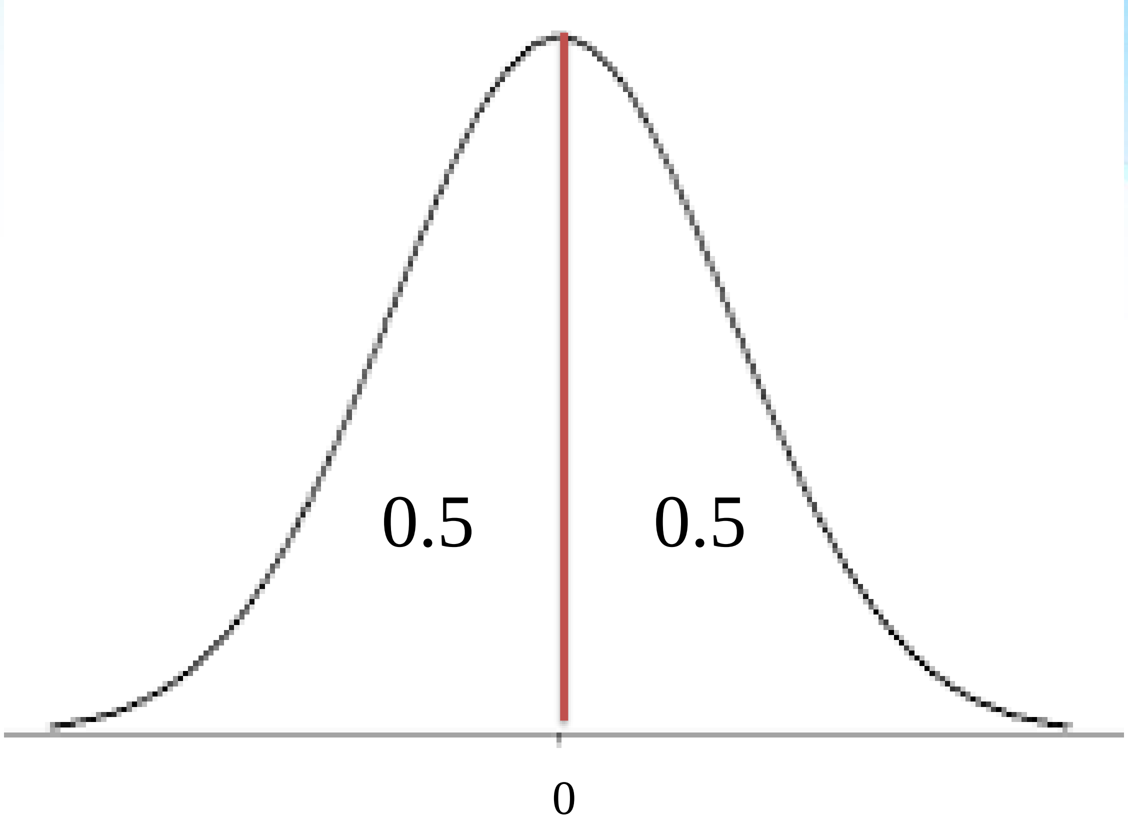
$$3)\ P(-1.33 \le Z \le 2.42) = P(Z \le 2.42) - P(Z \le -1.33)$$
$$= 0.9922 - 0.0918 = 0.9004$$

z	0.00	0.01	0.02	0.03	.....
⋮					
-1.3				0.0918	
⋮					
2.4			0.9922		
⋮					



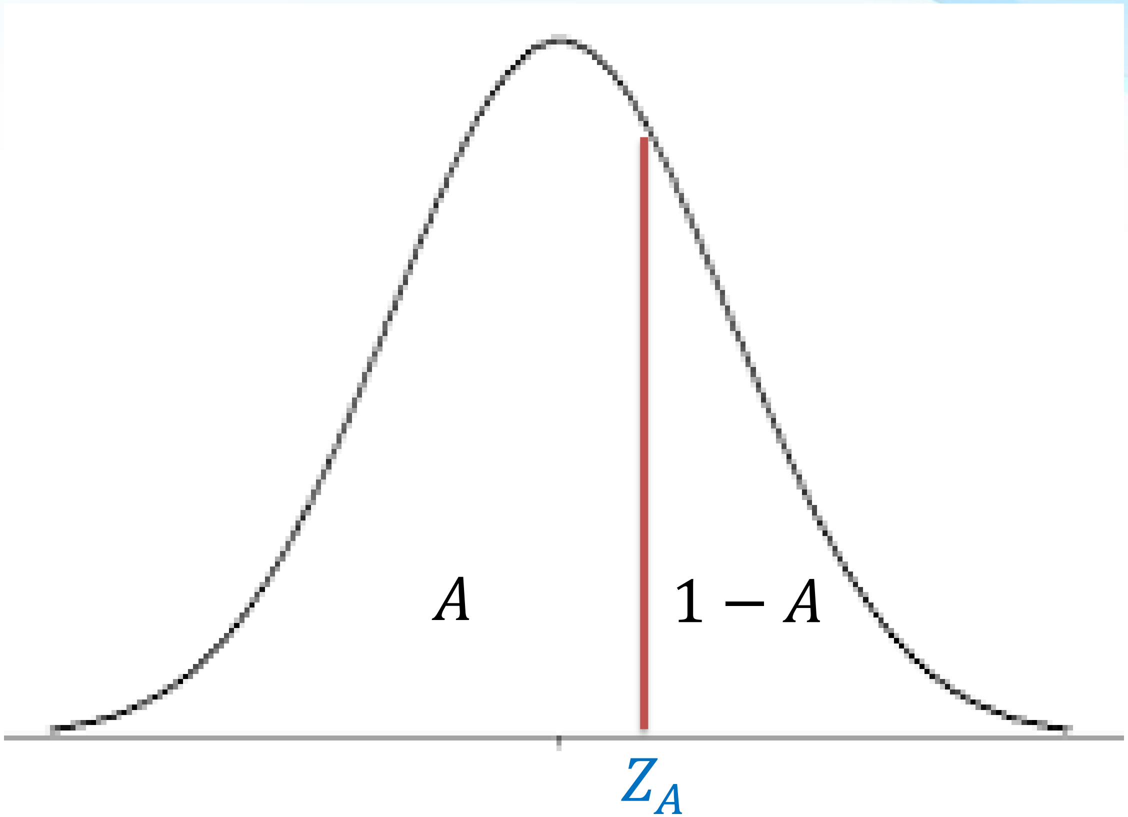
Example(4): $Z \sim N(0,1)$

4) $P(Z \geq 0) = P(Z \leq 0) = 0.5$



Notation

$$P(Z \leq Z_A) = A$$



Example: Z ~ N(0,1)

If $P(Z \le a) = 0.9505$

Z	…	0.05	…
:		$\Uparrow$	
1.6	$\Leftarrow$	0.9505	
:			

Then $a = 1.65$

Example: $Z \sim N(0,1)$

$$Z_{0.90} = 1.285$$

$$Z_{0.95} = 1.645$$

$$Z_{0.975} = 1.96$$

$$Z_{0.99} = 2.325$$

Calculating Probabilities of Normal (μ, σ^2)

$$X \sim \text{Normal}\left(\mu, \sigma^2\right) \Leftrightarrow Z = \frac{X-\mu}{\sigma} \sim \text{Normal}\left(0,1\right)$$

$$X \le a \Leftrightarrow \frac{X-\mu}{\sigma} \le \frac{a-\mu}{\sigma} \Leftrightarrow Z \le \frac{a-\mu}{\sigma}$$

(i) $P(X \le a) = P\left(Z \le \dfrac{a-\mu}{\sigma}\right)$

(ii) $P(X \ge a) = 1 - P(X \le a) = 1 - P\left(Z \le \dfrac{a-\mu}{\sigma}\right)$

(iii)

$$P(a \le X \le b) = P(X \le b) - P(X \le a) = P\left(Z \le \frac{b-\mu}{\sigma}\right) - P\left(Z \le \frac{a-\mu}{\sigma}\right)$$

(iv) $P(X = a) = 0$ for every a.

$(V)\ P(X \le \mu) = P(X \ge \mu) = 0.5$

Example:

Suppose that the hemoglobin level for healthy adults males has a normal distribution with mean $\mu=16$ and variance $\sigma^2=0.81$ (standard deviation $\sigma=0.9$).

(a) Find the probability that a randomly chosen healthy adult male has hemoglobin level less than 14.

(b) What is the percentage of healthy adult males who have hemoglobin level less than 14?

Solution:

(a) $P(X \leq 14) = P\left(Z \leq \frac{14-\mu}{\sigma}\right)$

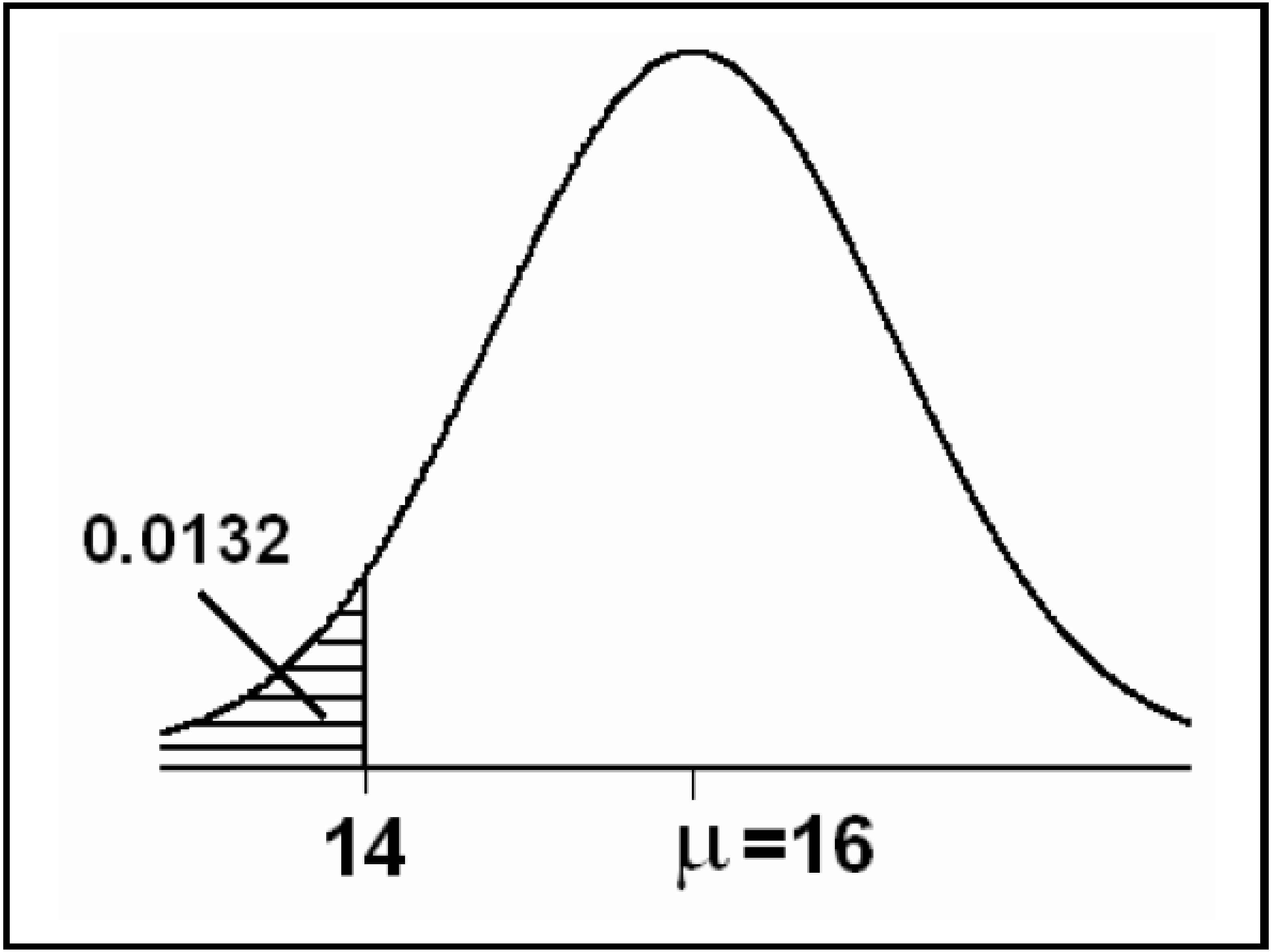
$= P\left(Z \leq \frac{14-16}{0.9}\right) = P(Z$

$\leq -2.22) = 0.0132$

(b) The percentage of healthy adult males who have hemoglobin level less than 14 is

P(X ≤14) × 100% = 0.01320× 100% =1.32%

1.32% of healthy adult males have hemoglobin level less than 14.


0.0132
14
μ =16

Example:

Suppose that the birth weight of Saudi babies has a normal distribution with mean $\mu=3.4$ and standard deviation $\sigma=0.35$.

(a) Find the probability that a randomly chosen Saudi baby has a birth weight between 3.0 and 4.0 kg.

(b) What is the percentage of Saudi babies who have a birth weight between 3.0 and 4.0 kg?

Solution:

(a) $$P(3.0 < X < 4.0) = P(X \le 4.0) - P(X \le 3.0)$$

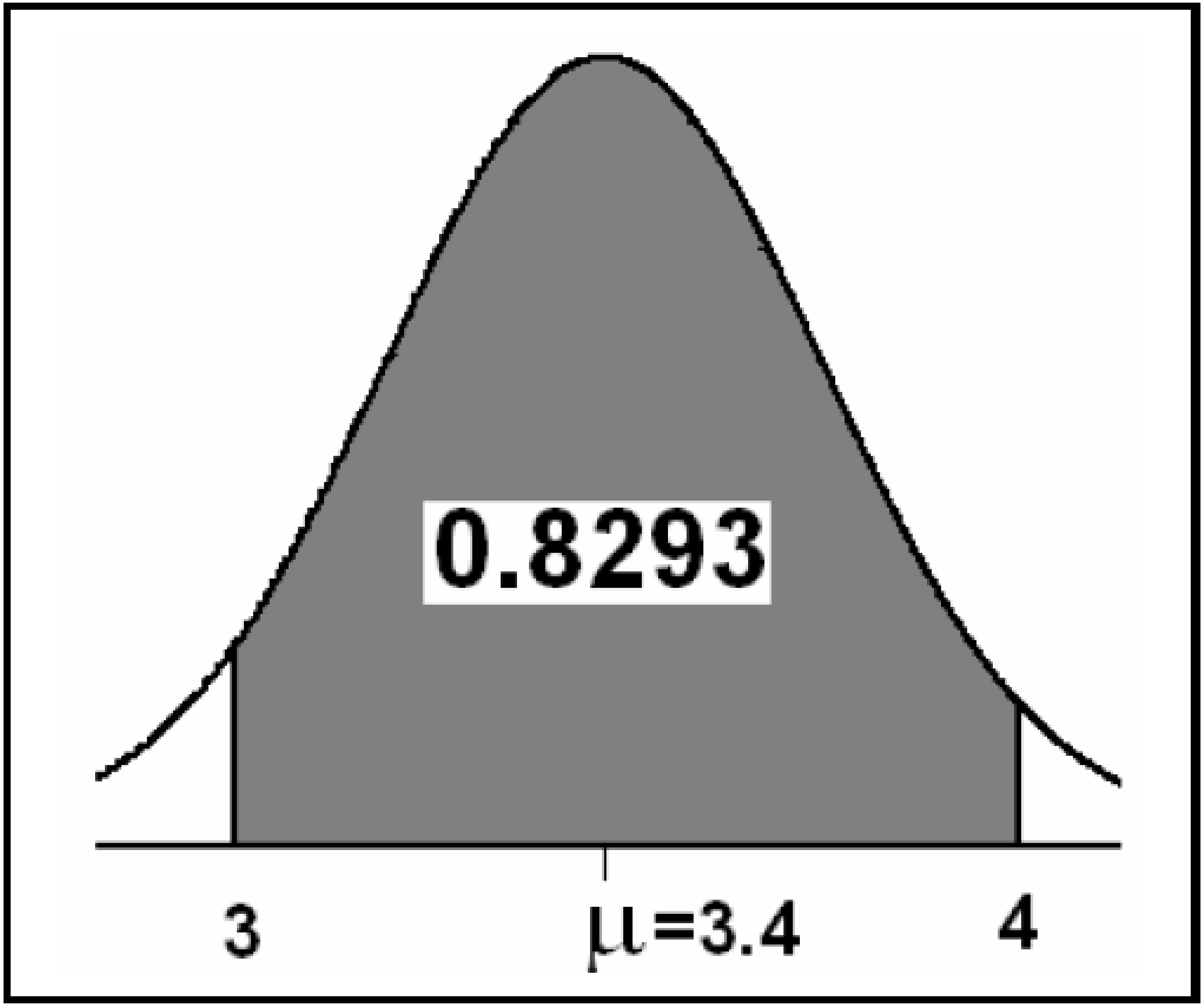
$$= P\left(Z \le \frac{4.0 - \mu}{\sigma}\right) - P\left(Z \le \frac{3.0 - \mu}{\sigma}\right)$$

$$= P\left(Z \le \frac{4.0 - 3.4}{0.35}\right) - P\left(Z \le \frac{3.0 - 3.4}{0.35}\right)$$

$$= P(Z \le 1.71) - P(Z \le -1.14)$$

$$= 0.9564 - 0.1271 = 0.8293$$

(b) 82.93% of Saudi babies have birth weight between 3.0 and 4.0 kg.


0.8293
3
μ=3.4
4

Notation:

$$P(Z \geq Z_A) = A$$

Result:

$$Z_A = -Z_{1-A}$$

Example:

$Z \sim N(0,1)$

$P(Z \geq Z_{0.025}) = 0.025$

$P(Z \geq Z_{0.95}) = 0.95$

$P(Z \geq Z_{0.90}) = 0.90$

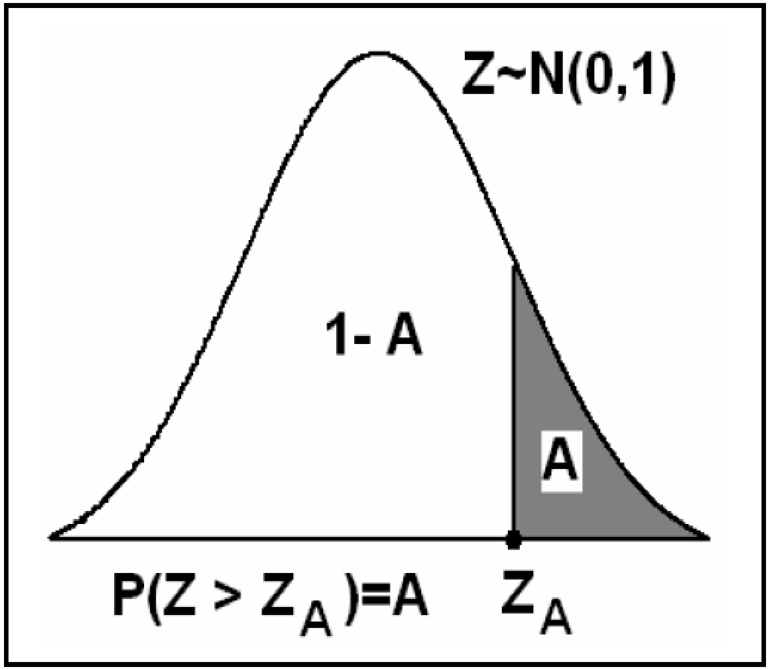
Example:

$Z \sim N(0,1)$

$Z_{0.025} = 1.96$

$Z_{0.95} = -1.645$

$Z_{0.90} = -1.285$



Z	…	0.06	
⋮	⋮	⇑ ⇑	
1.9	⇐⇐	0.975	

$P(Z \geq Z_{0.025}) = 0.025$

$Z_{0.025} = 1.96$

More examples see

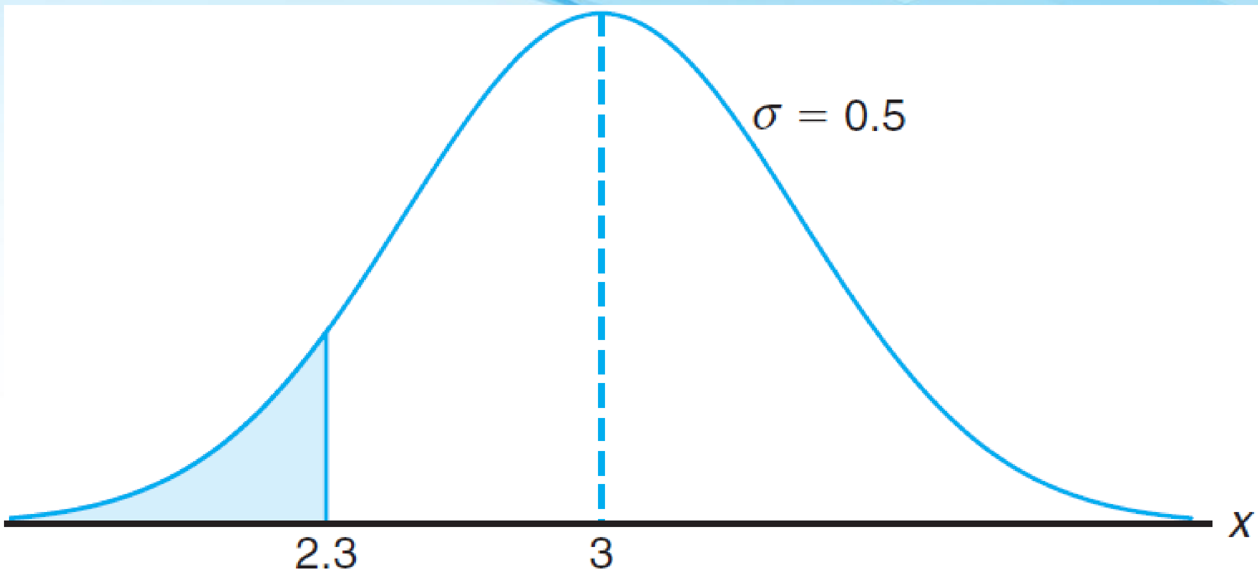
Ex: 6.2, 6.3, 6.4 and 6.5 on page 178

Applications of the Normal Distribution

Example:

A certain type of storage battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that battery life is normally distributed, find the probability that a given battery will last less than 2.3 years.

Solution:



$$z = \frac{2.3 - 3}{0.5} = -1.4,$$

$$P(X < 2.3) = P(Z < -1.4) = 0.0808.$$

Example

In an industrial process, the diameter of a ball bearing is an important component part. The buyer sets specifications on the diameter to be 3.00 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that, in the process, the diameter of a ball bearing has a normal distribution with mean 3.00 cm and standard deviation 0.005 cm. On the average, how many manufactured ball bearings will be scrapped?

Solution:

$\mu = 3.00$

$\sigma = 0.005$

X=diameter

$X \sim N(3.00, 0.005)$

The specification limits are:

3.00 ± 0.01

x_1=Lower limit$=3.00-0.01=2.99$

x_2=Upper limit$=3.00+0.01=3.01$

$$P(x_1 < X < x_2) = P(2.99 < X < 3.01) = P(X < 3.01) - P(X < 2.99)$$

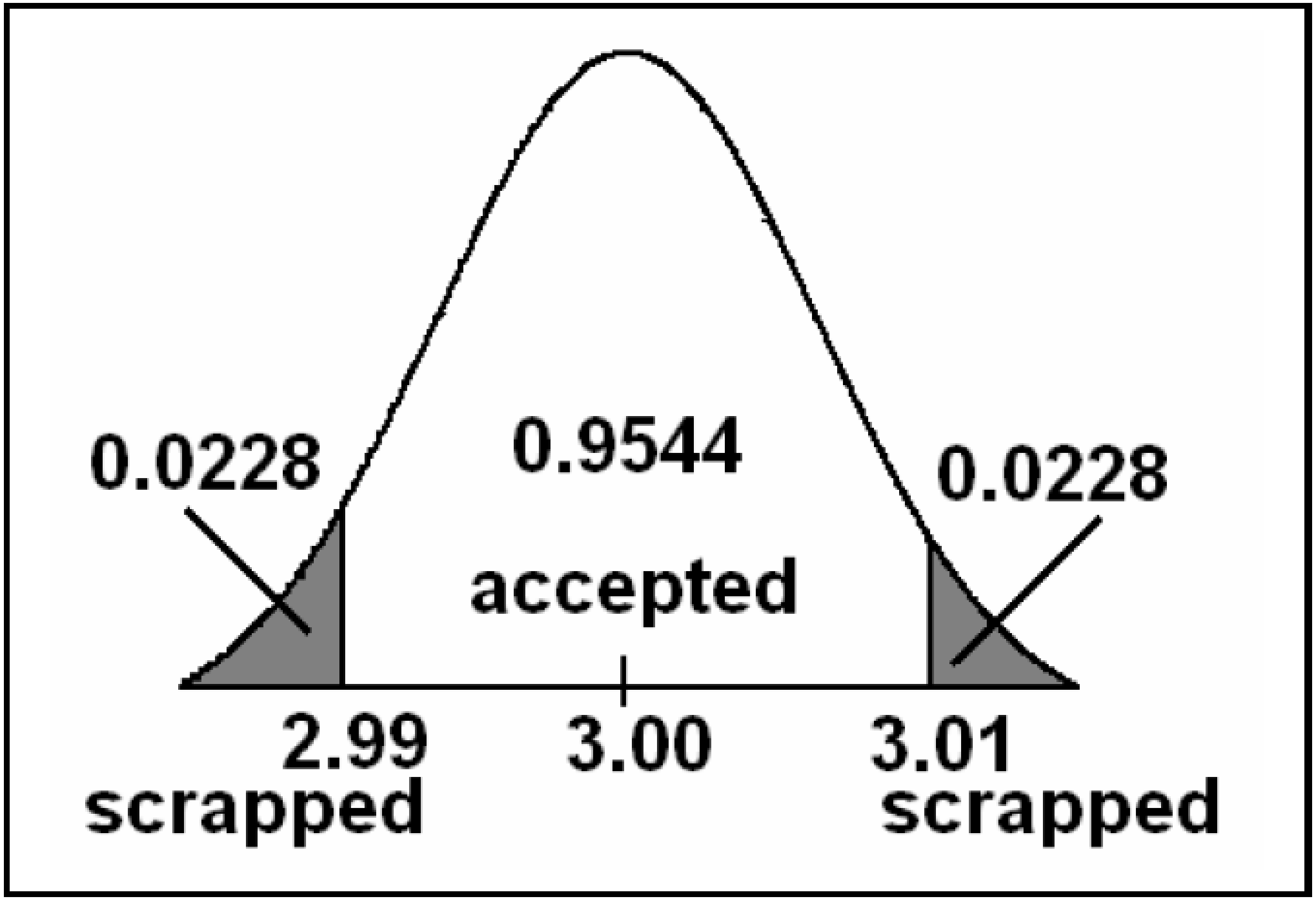
$$= P\left(Z \le \frac{3.01 - \mu}{\sigma}\right) - P\left(Z \le \frac{2.99 - \mu}{\sigma}\right)$$

$$= P\left(Z \le \frac{3.01 - 3.00}{0.005}\right) - P\left(Z \le \frac{2.99 - 3.00}{0.005}\right)$$

$$= P(Z \le 2.00) - P(Z \le -2.00)$$

$$= 0.9772 - 0.0228$$

$$= 0.9544$$



Therefore, on the average, 95.44% of manufactured ball bearings will be accepted and 4.56% will be scrapped.

Example

Gauges are used to reject all components where a certain dimension is not within the specifications $1.50 \pm d$. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 0.20. Determine the value d such that the specifications cover 95% of the measurements.

Solution:

$\mu = 1.5$

$\sigma = 0.20$

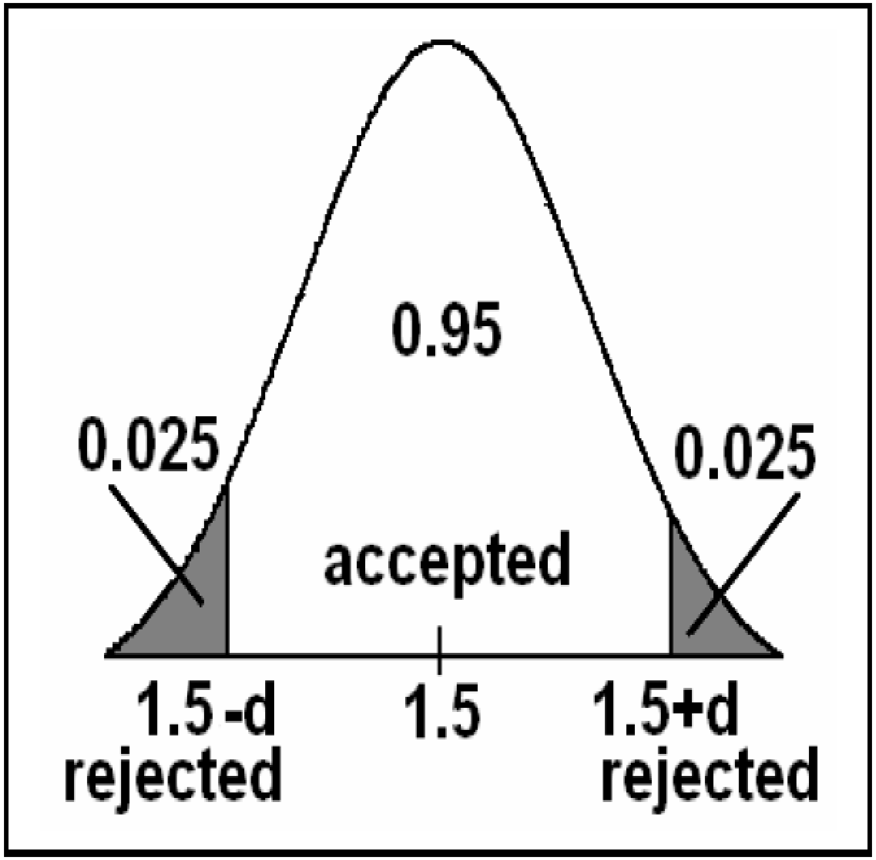
X= measurement

$X \sim N(1.5, 0.20)$

The specification limits are $1.5 \pm d$

x_1=Lower limit$=1.5-d$

x_2=Upper limit$=1.5+d$



$$P(X > 1.5 + d) = 0.025 \iff P(X < 1.5 + d) = 0.975$$

$$P(X < 1.5 - d) = 0.025$$

$$P(X < 1.5 - d) = 0.025$$

$$\iff P\left(\frac{X-\mu}{\sigma} \leq \frac{(1.5-d)-\mu}{\sigma}\right) = 0.025$$

$$\Leftrightarrow P\left(Z \le \frac{(1.5-d)-\mu}{\sigma}\right)=0.025$$

$$\Leftrightarrow P\left(Z \le \frac{(1.5-d)-1.5}{0.20}\right)=0.025$$

$$\Leftrightarrow P\left(Z \le \frac{-d}{0.20}\right)=0.025$$

$$\Leftrightarrow \frac{-d}{0.20} = -1.96$$

$$\Leftrightarrow -\mathrm{d}= (0.20)(-1.96)$$

$$\Leftrightarrow \mathrm{d}= 0.392$$

Z	…	0.06
⋮	⋮	$\Uparrow$ $\Uparrow$
-1.9	$\Leftarrow\Leftarrow$	0.025

$$P\left(Z \le \frac{-d}{0.20}\right) = 0.025$$

$$\frac{-d}{0.20} = -1.96$$

Note: $\frac{-d}{0.20} = Z_{0.025}$

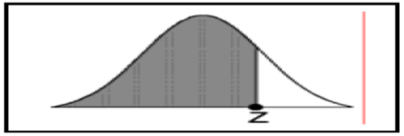
The specification limits are:

x_1=Lower limit=1.5−d = 1.5 − 0.392 = 1.108

x_2=Upper limit=1.5+d=1.5+0.392= 1.892

Therefore, 95% of the measurements fall within the specifications (1.108, 1.892).

TABLE:
Areas Under The Standard Normal Curve
Z~Normal(0,1)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998