1/ Lecture (5) Definite Integral by using Riemann Sum & Fundamental theorem of Calculus

* Revise Lecture (4)

EX (2) Evaluate

$$\int_{-2}^{7} (6-2x)\,dx \text{ by using Riemann sum}$$

Ans:

$$\therefore R_n = \sum_{k=1}^{n} f(x_k)\Delta x$$

$$\sum_{k=1}^{n} c = nc$$

$$\Delta x = \frac{b-a}{n} = \frac{7+2}{n} = \frac{9}{n},$$

$$x_k = a + k\Delta x = -2 + \frac{9k}{n}$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\therefore R_n = \sum_{k=1}^{n} \left(6+4-\frac{18k}{n}\right)\frac{9}{n}$$

$$R_n = \frac{9}{n}\sum_{k=1}^{n}\left(10-\frac{18k}{n}\right) = \frac{9}{n}\left[10n - \frac{\overset{9}{\cancel{18}}}{n}\frac{n(n+1)}{\cancel{2}}\right]$$

$$\therefore R_n = 90 - 81\left(\frac{n+1}{n}\right)$$

$$\therefore \int_{-2}^{7}(6-2x)\,dx = \lim_{n\to\infty} R_n = 90-81 = 9$$

$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

EX (3) Evaluate $\int_0^2 (6x^3-6x^2-1)\,dx$ by using Riemann Sum

Ans:

$$R_n = \sum_{k=1}^{n} f(x_k)\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n},$$

$$x_k = a + k\Delta x = \frac{2k}{n}$$

$$R_n = \sum_{k=1}^{n}\left(\frac{48k^3}{n^3} - \frac{24k^2}{n^2} - 1\right)\frac{2}{n}$$

$$R_n = \frac{96}{n^4}\sum_{k=1}^{n} k^3 - \frac{48}{n^3}\sum_{k=1}^{n} k^2 - \frac{2}{n}\sum_{k=1}^{n} 1$$

$$R_n = \frac{96}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{48}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] - \frac{2}{\cancel{n}}\cancel{n}$$

$$R_n = 24\left[\frac{n^2(n+1)^2}{n^4}\right] - 8\left[\frac{n(n+1)(2n+1)}{n^3}\right] - 2$$

$$\therefore \int_0^2 (6x^3-6x^2-1)\,dx = \lim_{n\to\infty} R_n$$

$$= 24 - 16 - 2 = 6$$

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2

The Fundamental theorem of calculus

- If f is continuous on $[a,b]$ and F is any antiderivative of f, then

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

- **Theorem**

$$\int_a^b f(x)\,dx = \Big[\int f(x)\,dx\Big]_a^b$$

$$\Rightarrow \int_a^b x^r\,dx = \Big[\frac{x^{r+1}}{r+1}\Big]_a^b, \quad r \neq -1$$

$$\int_a^b \sin x\,dx = \Big[-\cos x\Big]_a^b$$

$$\int_a^b \sec^2 x\,dx = \Big[\tan x\Big]_a^b$$

* Check the results that are obtained in EX ①, EX ② and EX ③ for "definite Integrals by Riemann".

EX ①

$$\int_0^4 (x+1)\,dx = \Big[\frac{x^2}{2} + x\Big]_0^4 = (8+4) - 0 = 12$$

EX ②

$$\int_{-2}^{7} (6-2x)\,dx = \Big[6x - \frac{2x^2}{2}\Big]_{-2}^{7}$$

$$= (42-49) - (-12-4) = -7 + 16 = 9$$

EX ③

$$\int_0^2 (6x^3 - 6x^2 - 1)\,dx$$

$$= \Big[\frac{6x^4}{4} - \frac{6x^3}{3} - x\Big]_0^2$$

$$= (24 - 16 - 2) - 0$$

$$= 6$$

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3

H.W

- Use Riemann sum to evaluate the following definite Integrals:

pb ① $\int_1^2 (6x-5)\,dx$

Ans: 4

pb ② $\int_0^2 3x^2\,dx$

Ans: 8

pb ③ $\int_1^2 (6-2x)\,dx$

Ans: 3

pb ④ $\int_1^3 (1-2x)\,dx$

Ans: -6

pb ⑤ $\int_0^2 (6x^3-1)\,dx$

Ans: 22

pb ⑥ $\int_0^2 (4x^3-3x^2-1)\,dx$

Ans: 6

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